MEAN

1.

Let the total average be t, percentage of director is d. Then,
t*100=(t-5000)(100-d)+(t+15000)d
d can be solve out.
Answer is C

2. Let x = the number of 20 oz. bottles
$48 - x$ = the number of 40 oz. bottles
The average volume of the 48 bottles in stock can be calculated as a weighted average:

$$\frac{x(20) + (48 - x)(40)}{48} = 35$$

So $x = 12$

Therefore there are 12 twenty oz. bottles and 48 – 12 = 36 forty oz. bottles in stock. If no twenty oz. bottles are to be sold, we can calculate the number of forty oz. bottles it would take to yield an average volume of 25 oz:
Let n = number of 40 oz. bottles

$$\frac{(12)(20) + (n)(40)}{n + 12} = 25$$

$(12)(20) + 40n = 25n + (12)(25)$
$15n = (12)(25) - (12)(20)$
$15n = (12)(25 - 20)$
$15n = (12)(5)$
$15n = 60$
$n = 4$
Since it would take 4 forty oz. bottles along with 12 twenty oz. bottles to yield an average volume of 25 oz, 36 – 4 = 32 forty oz. bottles must be sold. The correct answer is D.

3.

This question deals with weighted averages. A weighted average is used to combine the averages of two or more subgroups and to compute the overall average of a group. The two subgroups in this question are the men and women. Each subgroup has an average weight (the women's is given in the question; the men's is given in the first statement). To calculate the overall average weight of the group, we would need the averages of each subgroup along with the ratio of men to women. The ratio of men to women would determine the weight to give to each subgroup's

average. However, this question is not asking for the weighted average, but is simply asking for the ratio of women to men (i.e. what percentage of the competitors were women).
(1) INSUFFICIENT: This statement merely provides us with the average of the other subgroup – the men. We don't know what weight to give to either subgroup; therefore we don't know the ratio of the women to men.
(2) SUFFICIENT: If the average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women, there must be twice as many men as women. With a 2:1 ratio of men to women of, 33 1/3% (i.e. 1/3) of the competitors must have been women. Consider the following rule and its proof.
RULE: The ratio that determines how to weight the averages of two or more subgroups in a weighted average ALSO REFLECTS the ratio of the distances from the weighted average to each subgroup's average.
Let's use this question to understand what this rule means. If we start from the solution, we will see why this rule holds true. The average weight of the men here is 150 lbs, and the average weight of the women is 120 lbs. There are twice as many men as women in the group (from the solution) so to calculate the weighted average, we would use the formula [1(120) + 2(150)] / 3. If we do the math, the overall weighted average comes to 140.
Now let's look at the distance from the weighted average to the average of each subgroup.
Distance from the weighted avg. to the avg. weight of the men is 150 – 140 = 10.
Distance from the weighted avg. to the avg. weight of the women is 140 – 120 = 20.
Notice that the weighted average is twice as close to the men's average as it is to the women's average, and notice that this reflects the fact that there were twice as many men as women. In general, the ratio of these distances will always reflect the relative ratio of the subgroups.
The correct answer is (B), Statement (2) ALONE is sufficient to answer the question, but statement (1) alone is not.

4. The average number of vacation days taken this year can be calculated by dividing the total number of vacation days by the number of employees. Since we know the total number of employees, we can rephrase the question as: How many total vacation days did the employees of Company X take this year?
(1) INSUFFICIENT: Since we don't know the specific details of how many vacation days each employee took the year before, we cannot determine the actual numbers that a 50% increase or a 50% decrease represent. For example, a 50% increase for someone who took 40 vacation days last year is going to affect the overall average more than the same percentage increase for someone who took only 4 days of vacation last year.
(2) SUFFICIENT: If three employees took 10 more vacation days each, and two employees took 5 fewer vacation days each, then we can calculate how the number of vacation days taken this year differs from the number taken last year:
(10 more days/employee)(3 employees) – (5 fewer days/employee)(2 employees) = 30

days – 10 days = 20 days
20 additional vacation days were taken this year.
In order to determine the total number of vacation days taken this year (i.e., in order to answer the rephrased question), we need to determine the number of vacation days taken last year. The 5 employees took an average of 16 vacation days each last year, so the total number of vacation days taken last year can be determine by taking the product of the two:
(5 employees)(16 days/employee) = 80 days
80 vacation days were taken last year. Hence, **the total number of vacation days taken this year was 100 days.**
Note: It is not necessary to make the above calculations -- it is simply enough to know that you have enough information in order to do so (i.e., the information given is sufficient)! The correct answer is B.

5.

The formula for calculating the average (arithmetic mean) home sale price is as follows:

$$\text{Average} = \frac{\text{sum of home sale prices}}{\text{number of homes sold}}$$

A suitable rephrase of this question is "What was the sum of the homes sale prices, and how many homes were sold?"

(1) Seems SUFFICIENT as this statement tells us the sum of the home sale prices and the number of homes sold. Thus, the average home price is $51,000,000/100 = $510,000.
But we are not sure how many homes were there overall (may be more than 100). So INSUFFICIENT.

(2) INSUFFICIENT: This statement tells us the average condominium price, but not all of the homes sold in Greenville last July were condominiums. From this statement, we don't know anything about the other 40% of homes sold in Greenville, so we cannot calculate the average home sale price. Mathematically:

$$\text{Average} = \frac{\text{sum of condominium sale prices + sum of non-condominium sale prices}}{\text{number of condominiums sold + number of non-condominiums sold}}$$

We have some information about the ratio of number of condominiums to non-condominiums sold, 60%:40%, or 6:4, or 3:2, which could be used to pick working numbers for the total number of homes sold. However, the average still cannot be calculated because we don't have any information about the non-condominium prices.

The correct answer is E.

6.

The correct answer is C.
Lets say for X:

Number of members = n1
Sum of all the ages = s1
average ages = a1

Now for Y:

Number of members = n2
Sum of all the ages = s2
average ages = a2

And for Z:

Number of members = n1 + n2
Sum of all the ages = s1 + s2
average ages = a3

(1) The average (arithmetic mean) age of the members of Committee X is 25.7 years and the average age of the members of Committee Y is 29.3 years.
Which gives us a1 = 25.7 and a2 = 29.3
Also s1 = n1*25.7
and s2 = n2*29.3

Since we dont have any information on the rest this is INSUFFICIENT
(2) The average (arithmetic mean) age of the members of Committee Z will be 26.6 years
Which gives us a3 = 26.6
Still INSUFFICIENT

TOGETHER:

a3 = (s1+s2) / (n1+n2)

Fill in the values of a3, s1 and s2

26.6 = ((n1*25.7) + (n2*29.3)) / (n1+n2)

Simplify to get .9n1 = 2.7n2

or n1/n2 = 3/1 = 3

That means there are 3 times as many members in X (n1) as in Y (n2)

SUFFICIENT - anwser C

OR

in a WEIGHTED AVERAGE, if you have the 2 distances between the mean and each of the individual component averages, that's good enough to figure out the RATIO of the two components. here's how:

distance between committee A and average = 29.3 - 26.6 = 2.7
distance between committee B and average = 26.6 - 25.7 = 0.9

ratio = 3:1

committee B must be bigger (because the average is closer to committee B's average), so the ratio of committee B : committee A is 3:1 (not 1:3).

WARNING:
this procedure will NOT allow you to determine the actual SIZE of committee A or committee B, without any additional information; all it will give you is the RATIO of the two sizes. that's plenty sufficient to answer this problem, but, if the problem had asked for the NUMBER of members of any of the committees, the answer would be (e) as you had expected.

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btw, you don't have to do any of this ratio nonsense to solve this problem; it's sufficient to notice that the overall average is closer to the average for committee B than to that for committee A. if that's the case, then committee B must have a heavier influence on the average, a fact that means committee B has more people.

7.

so we know the average is x.
number of terms = 5
sum = 5x
since these are positive temps, no negative and zeros, the sum of the greatest 3 has to be less than 5x , also x has to be positive.
If you add the last 3 positive ##s the sum should be < sum of all 5
the only answer choice that fits is 4x, i.e B
all others choices are greater than 5x
next, you rule out (c), (d), and (e), for *exactly* the same reason you ruled out (a).
that leaves (b).
done.

8.

statement 1
1.15 (42.8) x 8
+
statement 2
(42.8) (2)
= the sum of the salaries.
That sum divided by 10 = the this year's avg.

Answer is E because given (1) and (2) together, we have no way of knowing the individual salaries and we cannot assume that the individual salaries are each equal to the previous year's average. For example, perhaps the highest 8 salaries increased by 15% and the lowest 2 did not change at all, the average would surely be different than if the highest 2 salaries did not change and the other 8 increased by 15%.

remember that you can, and should, work easily back and forth between the AVERAGE of a set of data and the SUM of those data. remember, Average x # of Data Points = Sum, so, for sets in which the # of data points is known (such as this one), **the SUM of the salaries will answer the data sufficiency problem just as well as the AVERAGE that's explicitly requested.**

this is an extremely valuable observation, because, while it's somewhat difficult to process changes to the *average* conceptually, it's splendidly easy to think about changes to the *sum*.

contrast the situation in which the 8 *lowest* salaries are augmented to that in which the 8 *highest* salaries are augmented.
since we're augmenting the salaries by a fixed *percentage*, it follows that the *absolute dollar changes* are smaller in the former case than in the latter case (because 15% of a smaller number is less than 15% of a larger number).
therefore, in the former case, the overall increase in the SUM will be smaller than it is in the latter case, because the individual salary changes are smaller.
that settles the issue; the sum could change by different amounts. therefore, the average could also change by different amounts. therefore, insufficient.

9.

if you want to *maximize* the number of books taken by one person, then you need to *minimize* the number of books taken by the other people.

incidentally, this is a really common theme in 'optimization' problems:
to maximize one quantity, minimize the others; to minimize one quantity, maximize the others.
this is supremely obvious in some circumstances - for instance, if a baseball team with a salary cap wants to pay superstar X as much as possible, it can only do so by paying all the other players as *little* as possible.

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another common theme:
if you're given a statement about an *average*, then you should transform it into a statement about a *sum*.

--

if we follow both of the above points of advice, we arrive at the following solution:
first, realize that the 'average of 2 books' statement is really just a roundabout way of telling you that the 30 students took out a total of 60 books.
the mentioned quantities add up to 32 books, so you have to account for the other 28 books, among 6 students.
if you *minimize* the book count for five of the six students, that's 3 books per student = 15 books.
28 - 15 = 13 books for the lucky sixth student.

10.

note that this is a WEIGHTED AVERAGES problem.
on weighted averages problems, if you know any 2 of the following 3, then you can find the third one:
1. the ratio of 'weights' of the different quantities
2. the values of the quantities
3. the weighted average

in this problem, we have the values of the quantities (36 and 50), as well as the weighted average (42). that's enough information to find the ratio of the 'weights' (which happens to be 4:3 m:f, although you don't particularly care because it's data sufficiency).
because you have a total for everybody (1400), that ratio is enough to give you a hard number for the # of women.
sufficient.

--

if you know the relationships posited above, you can get through weighted averages on data sufficiency very quickly - although it's still helpful to know the algebraic method so you can break it out on problem solving.

OR

stem: 42/100 x 1400 said that they considered engaging..., so 588 are considered engaging, 812 are not considered engaging...

2) of the 588, 288 are men, so 300 are women are engaging -- but how many women are

NOT considered engaging...?
INSUFFICIENT

harder of the 2 statements:

1) 36% of Men and 50% of Women are considered engaging...so $36/100 M + 50/100 W = 588$ and $M + W = 1400$

2 distinct linear equations, 2 variables
SUFFICIENT

answer is A

11.

Correct answer is B.

let there be x male students and y female students in the class. so the strength of the class is $x + y$.
our aim is to find out the ratio $x/(x+y)$

so $0.72x + 0.8y$ students applied colleges.

1. Is insufficient to find out the ratio mentioned above.

2. tells us that $0.72x + 0.8y = 0.75(x+y)$

solving the above equation we get $x/y = 5/3$
from this we can calculate the ratio $x/(x+y) = 5/8$.
So the answer is B.

12.

Incidentally, you can memorize the following fact: If you have the averages for two components of a group, and also a weighted average of the whole group, then you can find the ratio of the two components. This knowledge will in itself be sufficient to address many data sufficiency problems involving ratios and averages, without the rather extensive algebraic twists and turns shown here.

OR

Avg(arithematic mean) for Males = 9.8
Avg(arithematic mean) for females = 9.1

Statement (2) - Combined arithematic mean = 9.3

If you do the substitution, you get
(9.8x + 9.1y)/(x + y) = 9.3
multiply by x + y --> 9.8x + 9.1y = 9.3x + 9.3y
subtract 9.3x and 9.1y from both sides --> 0.5x = 0.2y
therefore x = (0.2/0.5)y
--> x = (2/5)y
x/y = 2/5

13.

Correct answer is A.

Note that, **if you have the averages of all the FRACTIONS or PERCENTAGES of a group, then you'll be able to calculate the overall average of the group**. This is a worthwhile fact to memorize for the data sufficiency problems.

As soon as you see A, you see that it is a classic weighted average. With weighted averages, to find the whole average, you do not need to know the total number of elements

A is saying { (n/3) (6 2.5) + (2n/3) (5 10) } / n

n's in numerator and denominator get canceled, in any weighted avg problem.

Looking at statement (2) first, we see that it is not sufficient, because the average (arithmetic mean) of a group of numbers is defined as (sum of data) / (# of data points). With statement (2), we only have the numerator of this expression (the # of people in the group is unknown), so we can't figure out the average.

Looking at statement (1) alone, we can set up the average as follows:
Average = (sum of data points) / (# of data points)
= [(n/3)(74.5) + (2n/3)(70)] / (n) <-- note that I used inches here, so I won't have to write in more fractions than necessary (trying to write fractions on this forum is not fun)
= [(1/3)(74.5) + (2/3)(70)] / (n)
There's no need to simplify further, because the 'n' is gone: you get one number. Therefore, this statement is sufficient.

Answer = A

14.

To find the average charge per hour, we need to know the total number of hours that the lawyer charged this particular client.
Now, total charge = $1550. From this we subtract $200, since that’s what the lawyer charges for the first hour.

Therefore, 1550 - 200 = 1350. Now divide this by 150 since the lawyer charges $150 for every hour after the first hour.

Therefore, 1350/150 = 9 hrs.

Add, the initial 1 hour. Therefore, the lawyer charged her client $1550 for 10 hours, which gives us an average of $155/hr. And that is option A.

15. A

16.

The only fundamental we need to remember is that for a equally spaced set , if the total number of items are odd then mean = median.

Consecutive numbers are a special case of equally spaced set.

So for 11 consecutive integers 6th number is the median so it will be the mean.

St1. the mean of first 9 integers is 7, which tells that the 5th number is 7, you know 5th number so you know 6th number is 8.

So avg of 11 numbers is 8.

SUFFICIENT.

St.2 In this list of 9 numbers mean is 9 so median = 9.
this is 7th number of original list so 6th number is 8. Which is the median = mean.

SUFFICIENT

Ans D.

The key to this problem is knowing that when we have an odd number of consecutive integers, the mean = median. So the question can be rephrased: "what is the 6th integer"?

Statement (1) is a set of the first nine integers, so the mean = median = 7 = 5th integer. Well, the 6th integer then is 8, so (1) is sufficient.

If eleven consecutive integers are listed from least to greatest, what is the average (arithmetic mean) of the eleven integers?

You can use the same reasoning to determine that (2) is also sufficient.

17.

the key to this question is **rephrasing**.
instead of the question that's actually posed here, you can ask the following question: **on which day did jane make the payment?**

there are two ways you can get this rephrase:
* use intuition
the later jane makes the payment, the higher the average daily balance on her account. therefore, any unique value for the account balance must correspond to a unique day on which the payment was made, and vice versa. so, knowing the day on which the payment was made is equivalent to knowing the average daily balance, and vice versa.
* use algebra
if jane makes the payment on day #n, then there are (n - 1) days for which the balance is $600, and 30 - (n- 1) = (31 - n) days for which the balance is $300. therefore, the average balance is (600(n - 1) + 300(29 -n)) / 30. this is a linear expression, so it can be solved uniquely for n. therefore, asking for the value of this expression is equivalent to asking for n itself.

so, there's the rephrase: **on which day did jane make the payment?**

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once you have this:

statement (1) is immediately sufficient; you don't even have to do anything.

statement (2) can be handled using exactly the same techniques used for the rephrase above:
* use intuition
the later jane makes the payment, the higher the average daily balance on her account. therefore, any unique value for the account balance must correspond to a unique day on which the payment was made, and vice versa. so, knowing the day on which the payment was made is equivalent to knowing the average daily balance, and vice versa. so, since you know the balance, you know on which day the payment was made, so this is sufficient.
* use algebra
if jane makes the payment on day #n, then there are (n - 1) days for which the balance is $600, and 25 - (n- 1) = (26 - n) days for which the balance is $300. therefore, the average balance is (600(n - 1) + 300(26 -n)) / 25. this is a linear expression, so it can be solved uniquely for n. therefore, if you are given the value of this expression, then, perforce, you also have the value of n..

so each statement is individually sufficient.

remember that this is data sufficiency, not problem solving. **you don't have to do the algebra**.
a big, big part of solving data sufficiency problems efficiently, especially data

sufficiency *word* problems, is realizing when you do and don't actually have to solve all the way through.

18. Answer B

you should be EXTREMELY wary of choosing (c) on questions like this one.

(c) is a "SUCKER ANSWER": they hand you so much information that you can figure out EVERY QUANTITY IN THE PROBLEM with NOTHING BUT SIMPLE ARITHMETIC.
i.e., if you have both of the statements together, then it's a simple matter to figure out that mary's salary is 50,000, giving you ALL of the salaries in the problem.

this is not the way DS problems usually work on the official test. **if the two choices together just HAND you all the information in the problem - with little to no work, apart from simple arithmetic - then the answer is most likely NOT (c).**
we've dealt with this elsewhere in this forum; do a search for the term "c trap" to find that treatment.

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in this case, the key is to figure out that **the salaries are EVENLY SPACED**. i.e., the distance between jim's and kate's salaries is the same as the distance between kate's and mary's.
i.e., on a number line:
<----------jim----------kate----------mary---------->
(where the blue distances are the same)

remember that, **in equally spaced sets, the MEAN is equal to the MEDIAN.**
therefore, the middle value is sufficient to answer the problem.

(1) doesn't give the middle value, so (1) is insufficient.

(2) gives the middle value, so (2) is sufficient.

ans = (b)

MEDIAN:

1-E

2

(1) These data will produce an average of = 72˚ for last April in City X. However, there is no information regarding the median for comparison; NOT sufficient.
(2) The median is the middle temperature of the data. As such, 50 percent of the daily high temperatures will be at or above the median, and 50 percent will be at or below the median. If 60 percent of the daily high temperatures were less than the average daily high temperature, then the average of the daily highs must be greater than the median; SUFFICIENT. The correct answer is B; statement 2 alone is sufficient.

3.

(1) Although 25 percent of the projects have 4 or more employees, there is essentially no information about the middle values of the numbers of employees per project. For example, if there were a total of 100 projects, then the median could be 2 (75 projects that have exactly 2 employees each and 25 projects that have exactly 4 employees each) or the median could be 3 (75 projects that have exactly 3 employees each and 25 projects that have exactly 4 employees each); NOT sufficient.
(2) Although 35 percent of the projects have 2 or fewer employees, there is essentially no information about the middle values of the numbers of employees per project. For example, if there were a total of 100 projects, then the median could be 3 (35 projects that have exactly 2 employees each and 65 projects that have exactly 3 employees each) or the median could be 4 (35 projects that have exactly 2 employees each and 65 projects that have exactly 4 employees each); NOT sufficient. Given both (1) and (2), 100 – (25 + 35) percent = 40 percent of the projects have exactly 3 employees. Therefore, when the numbers of employees per project are listed from least to greatest, 35 percent of the numbers are 2 or less and (35 + 40) percent = 75 percent are 3 or less, and hence the median is 3. The correct answer is C; both statements together are sufficient.

4.

(1) The information given says nothing about the number of books on the lower shelf. If there are fewer than 25 books on the lower shelf, then the median number of pages will be the number of pages in one of the books on the upper shelf or the average number of pages in two books on the upper shelf. Hence, the median will be at most 400. If there are more than 25 books on the lower shelf, then the median number of pages will be the number of pages in one of the books on the lower shelf or the average number of pages in two books on the lower shelf. Hence, the median will be at least 475; NOT sufficient.
(2) An analysis very similar to that used in (1) shows the information given is not sufficient to determine the median; NOT sufficient. Given both (1) and (2), it follows that there is a total of 49 books. Therefore, the median will be the 25th book when the books are ordered by number of pages. Since the 25th book in this ordering is the book on the upper shelf with the greatest number of pages, the median is 400. Therefore, (1) and (2) together are sufficient. The correct answer is C; both statements together are sufficient.

5.

Given the list k, n, 12, 6, 17, determine the value of n.

(1) Although k < n, no information is given about the value of k or n; NOT sufficient.

(2) Since the median of the numbers in the list is 10 and there are 5 numbers in the list, 10 is one of those 5 numbers. Therefore, n = 10 or k = 10. If n = 10, then the value of n has been determined. However, if k = 10, then n can be any number that is 10 or less, so the value of n cannot be determined; NOT sufficient.

Taking (1) and (2) together, if k < n and the median of the list is 10, then 12 and 17 are to the right of the median and the list in ascending order is either 6, k, n, 12, 17 or k, 6, n, 12, 17. In either case, n is the middle number, and since the median is 10, n = 10; SUFFICIENT. The correct answer is C; both statements together are sufficient.

6.

Let T, J, and S be the purchase prices for Tom's, Jane's, and Sue's new houses. Given that the average purchase price is 120,000, or T + J + S = (3)(120,000), determine the median purchase price.
(1) Given T = 110,000, the median could be 120,000 (if J = 120,000 and S = 130,000) or 125,000 (if J = 125,000 and S = 125,000); NOT sufficient.
(2) Given J = 120,000, the following two cases include every possibility consistent with T + J + S = (3)(120,000), or T + S = (2)(120,000). (i) T = S = 120,000 (ii) One of T or S is less than 120,000 and the other is greater than 120,000. In each case, the median is clearly 120,000; SUFFICIENT. The correct answer is B; statement 2 alone is sufficient.

7.

Since *S* contains only consecutive integers, its median is the average of the extreme values *a* and *b*. We also know that the median of *S* is $\frac{3}{4}b$. We can set up and simplify the following equation:

$$\frac{a+b}{2} = \frac{3b}{4} \rightarrow$$
$$4a + 4b = 6b \rightarrow$$
$$4a = 2b \rightarrow$$
$$2a = b$$

Since set *Q* contains only consecutive integers, its median is also the average of the extreme values, in this case *b* and *c*. We also know that the median of *Q* is $\frac{7}{8}c$. We can set up and simplify the following equation:

$$\frac{b+c}{2} = \frac{7c}{8} \rightarrow$$
$$8b + 8c = 14c \rightarrow$$
$$8b = 6c \rightarrow$$
$$4b = 3c$$

We can find the ratio of a to c as follows: Taking the first equation, $2a = b \rightarrow 8a = 4b$

and the second equation, $4b = 3c$ and setting them equal to each other, yields the following:

$8a = 3c \rightarrow \frac{a}{c} = \frac{3}{8}$. Since set R contains only consecutive integers, its median is the average of the extreme values a and c: $\frac{a+c}{2}$. We can use the ratio $\frac{a}{c} = \frac{3}{8}$ to substitute $\frac{3c}{8}$ for a:

$$\frac{\frac{3c}{8} + c}{2} \rightarrow$$
$$\frac{\frac{11c}{8}}{2} \rightarrow$$
$$\frac{11}{16}c$$

Thus the median of set R is $\frac{11}{16}c$. The correct answer is C.

8.

The set $R_n = R_{n-1} + 3$ describes an evenly spaced set: each value is three more than the previous. For example the set could be 3, 6, 9, 12 . . . For any evenly spaced set, the mean of the set is always equal to the median. A set of consecutive integers is an example of an evenly spaced set. If we find the mean of this set, we will be able to find the median because they are the same. (1) INSUFFICIENT: This does not give us any information about the value of the mean. The only other way to find the median of a set is to know every term of the set. (2) SUFFICIENT: The mean must be the median of the set since this is an evenly spaced set. This statement tells us that mean is 36. Therefore, the median must be 36. The correct answer is B.

9.

This question is asking us to find the median of the three scores. It may seem that the only way to do this is to find the value of each of the three scores, with the middle value taken as the median. Using both statements, we would have two of the three scores, along with the mean given in the question, so we would be able to find the value of the third score. It would seem then that the answer is C. On GMAT data sufficiency, always be suspicious, however, of such an obvious C. In such cases, one or both of the statements is often sufficient. (1) INSUFFICIENT: With an arithmetic mean of 78, the sum of the three scores is $3 \times 78 = 234$. If Peter scored 73, the other two scores must sum to $234 - 73 = 161$. We could come up with hundreds of sets of scores that fit these conditions and that have different medians. An example of just two sets are:
73, 80, 81 median = 80 61, 73, 100 median = 73 (2) SUFFICIENT: On the surface, this statement seems parallel to statement (1) and should therefore also be insufficient. However, we aren't just given one of the three scores in this statement. We are given a score with a value that is THE SAME AS THE MEAN. Conceptually, the mean is the point where the deviations of all the data net zero. This means that the sum of the differences from the mean to each of the points of data must net to zero. For a simple example, consider 11, which is the mean of 7, 10 and 16. $7 - 11 = -4$ (defined as negative because it is left of the mean on the number line) $10 - 11 = -1$ $16 - 11 = +5$ (defined as positive because it is right of the mean on the number line) The positive and negative deviations (differences from the mean) net to zero. In the question, we are told that the mean score is 78 and that Mary scored a 78. Mary's deviation then is $78 - 78 = 0$. For the deviations to net to zero, Peter and Paul's deviations must be $-x$ and $+x$ (not necessarily in that order).
Mary's deviation = $78 - 78 = 0$ Peter's (or Paul's) deviation = $-x$ Paul's (or Peter's) deviation = $+x$
We can then list the data in order: $78 - x$, 78, $78 + x$ This means that the median must be 78.
NOTE: x could be 0, which would simply mean that all three students scored a 78. However, the median would remain 78.
The correct answer is B.

10.

In order to solve the question easier, we simplify the numbers such as 150, 000 to 15, 130,000 to 13, and so on.
I. Median is 13, so, the greatest possible value of sum of eight prices that no more than median is 13*8=104. Therefore, the least value of sum of other seven homes that greater than median is (15*15-104)/7=17.3>16.5. It's true.
II. According the analysis above, the price could be, 13, 13, 13, 13, 13,13,13,13, 17.3, 17.3, 17.3... So, II is false.
III. Also false.
Answer: only I must be true.

11.

There are 73 scores, so, (73+1)/2=37, the 37th number is the median. It is contained by interval 80-89.
Answer is C

12.

Ann's actual sale is 450-x, Cal's 190+x, after corrected, Ann still higher than Cal, so Ann is the median.
Or we can explain it in another way:
450-x=330, so x=120
Ann's actual sale is 450-x Cal's 190+x,
Suppose that either Ann or Cal can be the median, if Ann is the median, than we get the previous answer; however, if Cal is median (330), we will have 190+x=330, x=140, then Ann(450-140=310) will less than Cal(330), that is incorrect.
This can explain why Cal can not be the median and Ann must

13.

To find the mean of the set {6, 7, 1, 5, *x*, *y*}, use the average formula: $A = \frac{S}{n}$ where *A* = the average, *S* = the sum of the terms, and *n* = the number of terms in the set.
Using the information given in statement (1) that *x* + *y* = 7, we can find the mean: $\frac{6+7+1+5+(x+y)}{6} = \frac{6+7+1+5+7}{6} = 4\frac{1}{3}$. Regardless of the values of *x* and *y*, the mean of the set is $4\frac{1}{3}$ because the sum of *x* and *y* does not change. To find the median, list the possible values for *x* and *y* such that *x* + *y* = 7. For each case, we can calculate the median.

x	*y*	DATA SET	MEDIAN
1	6	1, 1, 5, 6, 6, 7	5.5
2	5	1, 2, 5, 5, 6, 7	5
3	4	1, 3, 4, 5, 6, 7	4.5
4	3	1, 3, 4, 5, 6, 7	4.5
5	2	1, 2, 5, 5, 6, 7	5
6	1	1, 1, 5, 6, 6, 7	5.5

Regardless of the values of *x* and *y*, the median (4.5, 5, or 5.5) is always greater than the mean ($4\frac{1}{3}$). Therefore, statement (1) alone is sufficient to answer the question. Now consider statement (2). Because the sum of *x* and *y* is not fixed, the mean of the set will vary. Additionally, since there are many possible values for *x* and *y*, there are numerous possible medians. The following table illustrates that we can construct a data set for which $x - y = 3$ and the *mean* is greater than the median. The table ALSO shows that we can construct a data set for which $x - y = 3$ and the *median* is greater than the mean.

x	y	DATA SET	MEDIAN	MEAN
22	19	1, 5, 6, 7, 19, 22	6.5	10
4	1	1, 1, 4, 5, 6, 7	4.5	4

Thus, statement (2) alone is not sufficient to determine whether the mean is greater than the median. The correct answer is (A): Statement (1) alone is sufficient, but statement (2) alone is not sufficient.

14.

The total weight of X and Y to be distributed is 12. So, you want to min Y and max X. Y cannot be less than 9 because if it were, Y would change the median of 9. Hence , Y is 9 and X is 3.

15.

i) is insufficient. there is not enough information on the values of x,y,z.
ii) is sufficient. there are 3 possibilities: x<y=z, x=y=z, y=z<x. in any case z is the median.

Hence the answer is B

if you need convincing about answer a, then remember that your goal on these types of problems is to try to prove 'maybe' (i.e., insufficient). so try to find 2 groups of numbers, one of which gives a 'yes' answer and one of which gives a 'no' answer:
(x, y, z) = (1, 3, 5): z is not the median
(x, y, z) = (1, 5, 3): z is the median
insufficient.

16.

M is a +ve Odd Integer {1,3,5,7,9,11...} What is Avg of {M}?
Rephrase the Question as What is M?

ADBCE Grid:-
1) M:- {1,3,5,7,9,11..}
M:-{3,6} {3,6,9,12,15}.... Insuff to Find the Average

2) Median of set is 33. Insuff

Using 1 and 2
M(3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,....66,69}
Median is 33.
Since M is +ve odd integer,M should be odd.
If M is odd, then the middle integer will be the median

So if Median is 33, then M should be 21.
{3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,60,63}

Hence C

OR
(1) Insufficient. No way to know the mean. Consider one set {3, 6, 9} and another {6, 9, 12}.
(2) Insufficient. Knowing that there is an odd number of terms in the set and that the median is 33 does not tell us what the mean is.
(1&2) Sufficient. In an ordered set with an odd number of terms, the median is equal to the "middle" term. Moreover, in an equally distributed set of integers (like this one... consecutive multiples of 3) the median will equal the mean itself.

17.

(1)
x must be greater than 1 or 3. if it is 8 or less, then it will be the median.
therefore:
if x = 7, then the list is 1, 3, 7, 8, 12, and median = 7. average = 31/5 = 6.2; answer to question is YES
if x = 8, then the list is 1, 3, 8, 8, 12, and median = 8. average = 32/5 = 6.4; answer to question is YES
if x = REALLY BIG, then the list is 1, 3, 8, 12, x, and median = 8. average is REALLY BIG, so the answer to the question is NO.
insufficient

(2)
this means that x is at least 9, from the observations above.
the list is either 1, 3, 8, x, 12, or 1, 3, 8, 12, x, depending on the size of x.
regardless of which one is the proper ordering, the expressions for the median and the mean are the same:
median = 8
mean (average) = (1 + 3 + 8 + 12 + x)/5 = (24 + x)/5
since x is at least 9, the mean is at least (24 + 9)/5 = 33/5 = 6.6.
this is inconclusive, because the mean could be 6.6 (if x = 9) or could be in the millions (if x is huge).
insufficient

(together)
this doesn't help, because statement (2) by itself already implies that $x > 6$
so, still insufficient

answer = e

18.

this is another problem about a topic that is one of the darlings of the test authors: namely, sets of consecutive integers, especially as pertaining to the averages of such sets.

here is the fact that you absolutely must know about these sets:
the **mean** and the **median** of a set of consecutive integers are **the same**; both of them are the middle number (for a set with an odd # of numbers in it) or halfway between the two middle numbers (for a set with an even # of numbers in it).

--

remember that average = sum / number of data points. you should be ultra-aware of this relationship; the vast majority of problems about the sum of a set are really concerned with the average - and vice versa. they are tricky, those test writers.

let 'X' stand for the sum of each of the sets.

(1) is clearly insufficient, as we know nothing whatsoever about set t.
still, take the time to interpret it: it says that the middle number of set s is 0, which also means that the sumof the elements in set s is 0 (by the fact above).

(2)
using the fact above, we have that the average (whether mean or median - they're the same) of the numbers in set s is $X/5$, and the average (again, mean or median) of the numbers in set t is $X/7$.
it's tempting to say 'sufficient' here, because at first glance $X/5$ and $X/7$ appear to be necessarily different, but they aren't: in the singular case $X = 0$, the two will be identical.
therefore, insufficient.

(together)
this tells us that $X = 0$, which means that the median of both sets is $0/7 = 0/5 = 0$.
sufficient.

"the sum of these two sets(sets of consecutive integers) will be equal only when the sum is zero. " is **INCORRECT LOGIC**.
Ex: Set S could be 5,6,7,8 & 9 while Set T could be 2,3,4,5.6.7 & 8 and these sets have equal sums.
There are umpteen other examples.

19.

lesson one:
READ PROBLEMS CAREFULLY.
don't miss words like 'positive'. this is not a hard problem, but it's easy to miss if you are inattentive.

lesson two:
KNOW YOUR TERMINOLOGY / CLASSIFICATIONS.
zero is not a positive number.

the positive integers in the list are 1, 2, 3, 4, 5, 6, and 7, so the range is biggest - smallest = 7 - 1 = 6.

20.

to get at the heart of this problem, you must realize that "greater" is NOT the same thing as "farther away from zero".
for positive numbers, these two concepts are the same, but they are not so for negative numbers (for which the the greater number is actually closer to zero).
that realization is the crux of this problem.

the SUM is the same for each of the two sets. therefore, since average = (sum) / (# of data points), the average will be CLOSER TO ZERO if there are more data points.

the problem is that this doesn't mean that the average is lower. if the sum is negative, then just the opposite will occur.

examples:
if S = 2, 2, 2 and T = 3, 3, then the sums are both 6, the average of S is less (2 vs. 3), and S has more integers.
if S = -3, -3 and T = -2, -2, -2, then the sums are both -6, the average of S is less (-3 vs. -2), and S has fewer integers.
so (a) is insufficient.

if T = 2, 2, 2 and S = 3, 3, then the sums are both 6, the median of S is greater (3 vs. 2), and S has fewer integers.
if T = -3, -3 and S = -2, -2, -2, then the sums are both -6, the median of S is greater (-2 vs. -3), and S has more integers.
so (b) is insufficient.

together:
this takes a little more creativity.
if S = -7, 9, 10 and T = 6, 6, then the sums are both 12, the average of S is less (4 vs. 12), the

median of S is greater (9 vs. 6), and S has more integers.
if S = -6, -6 and T = -10, -9, 7, then the nums are both -12, the average of S is less (-6 vs. -4), the median of S is greater (-6 vs. -9), and S has fewer integers.
so, insufficient.

answer = (e).

incidentally, if this problem is supposed to say that the integers are positive or non-negative , then the problem is much easier, and the answer is different: (a).

Average = Sum/(numbers in set)

Sum of numbers in S = Ave(S) * (number of numbers in set S)
Sum of numbers in T = Ave(T) * (number of numbers in set T)

It is given that Sum(S)=Sum(t).

Thus,
Ave(S) * (number of numbers in set S) = Ave(T) * (number of numbers in set T)

If Ave(s)<Ave(t), then
(number of numbers in set S) > (number of numbers in set T).

21.

there are seven terms before a8, and seven terms after it. therefore, it's *the* middle term.
if you draw a line between a8 and a9, there will be eight terms to the left of the line and seven terms to the right of the line. therefore, that line is not the middle of the sequence.

--

general facts: (let *n* stand for the number of terms)

* *if you have an even # of terms*
in this case, the 'middle term' (median) is the average of the two middle terms, which are term number (n/2) and term number (n/2 + 1).

* *if you have an odd # of terms*
in this case, the middle term is just one term: term number (n + 1)/2.

The answer is (B)

If a(n) = a(n-1) + k, then a(1), a(2),.....a(15) is an arithmetic progression.

Statement (2) says a(8) = 10

a(8) is the middle term and hence the Median of this series. The number k can be positive or negative. If k is negative, then the numbers a(1), a(2) etc. will be in descending order and if it is positive, then the series will be in ascending order. No matter what, there will be 7 terms which will be greater than 10 (Please note that since k is non-zero, and hence all the terms in the series will be DISTINCT).

It could a(1) to a(7) if k is negative, and a(9) to a(15) if k is positive.

22.

OA:D

let's look at specific numbers. remember, when you pick numbers, you should TRY FOR INSUFFICIENT.
since this is a yes/no question, that means that you should try to get at least one "yes" and at least one "no".

it's easy to get a "yes": just take any equally spaced list, such as
68 69 70 71 72
or
50 60 70 80 90

so we're trying to get a "no".
* if we can find a list of 5 different numbers whose average is 70, and whose HIGHEST two numbers average 70 or less, then, insufficient.
* if we can't, then, sufficient.

let's try to find such a list.
since the average is 70, the sum must be 350.
now remember, the average of the HIGHEST two numbers must be 70 or less to get a "no".
this means that:
- the sum of those two numbers is 140 or less, AND
- the lower of the two (i.e., the second-highest number out of the five) is LESS than 70.
...but then the three other numbers, which are even lower, are also less than 70 each.
so we have (at most 140) + 3*(less than 70)
this is less than 350. that's a contradiction (the sum is supposed to be 350), so there can be no such set.

therefore, SUFFICIENT

--
finally, note that the condition stipulating that the numbers be DIFFERENT is crucially

important.
if you remove that condition, then the answer goes all the way to (e), since you can now use the set 70, 70, 70, 70, 70 to achieve a "no" even with both statements.

23.

If 5 pieces of wood have an average length of 124cm then the sum of the 5 pieces of wood must be 620.

Now if the median of the set is 140cm then lets imagine the following set based on the property of the median being the middle term of a set if the set is arranged in ascending order of length of wood.

[x, y, 140, 140, 140]

In order to maximise the length of the smallest segment we assume in the above set that the 4th and 5th largest pieces of wood are also 140cm as this does not change the median and still keeps our set in ascending order.

Hence x + y = 620 - 420 = 200

Now x<=y to keep the ascending order of our set so in order to maximise x, x must be equal to y.

Hence 2x = 200 and x can be 100cm at the outside and the set looks as follows:

[100, 100, 140, 140, 140]

24.

This is a weighted average problem:

90 (80/180) + 81 (100/180) =85

We know that her grade falls in the 81st percentile within the other class, because within that class of 100, 19 were ahead of her.

Amy's grade is 90th percentile, meaning that she outperformed 90% of her classmates.
That means that she outperformed 72 of her classmates.
She also outperformed 81 of the students in the other class (note: not 80, because Amy herself is not in the other class).

This means that she outperformed 72 + 81 = 153 of the combined 180 students in the two classes.

Since 153/180 = 0.85, she outperformed 85% of the students in the combined pool, putting her in the overall 85th percentile.

Range

1.

The range of a set of integers is equal to the difference between the largest integer and the smallest integer. The range of the set of integers 3, 4, 5, and 6 is 3, which is derived from 6 – 3.

(1) Although it is known that $y > 3x$, the value of x is unknown. If, for example, $x = 1$, then the value of y would be greater than 3. However, if $x = 2$, then the value of y would be greater than 6, and, since 6 would no longer be the largest integer, the range would be affected. Because the actual values of x and y are unknown, the value of the range is also unknown; NOT sufficient.

(2) If $x > 3$, and $y > x$, then x could be 4 and y could be 5. Then the range of the 6 integers would still be 6 – 3 or 3. However, if x were 4 and y were 15, then the range of the 6 integers would be 15 – 3, or 12. Th ere is no means to establish the values of x and y, beyond the fact that they both are greater than 3; NOT sufficient. Taking (1) and (2) together, it is known that $x > 3$ and that $y > 3x$. Since the smallest integer that x could be is thus 4, then $y > 3(4)$ or $y > 12$. Therefore, the integer y must be 13 or larger. When y is equal to 13, the range of the 6 integers is 13 – 3 = 10, which is larger than 9. As y increases in value, the value of the range will also increase. The correct answer is C; both statements together are sufficient.

2.

(1) If Carl began making $50 withdrawals on or before May 15, his account balance on April 16 would be at least $50 greater than it was on the last day of May. Thus, his account balance on April 16 would be at least $2,600 + $50 = $2,650, which is contrary to the information given in (1). Therefore, Carl did not begin making $50 withdrawals until June 15 or later. These observations can be used to give at least two possible ranges. Carl could have had an account balance of $2,000 on January 1, made $120 deposits in each of the first 11 months of the year, and then made a $50 withdrawal on December 15, which gives a range of monthly closing balances of (120)(10). Also, Carl could have had an account balance of $2,000 on January 1, made $120 deposits in each of the first 10 months of the year, and then made $50 withdrawals on November 15 and on December 15, which gives a range of monthly closing balances of (120)(9); NOT sufficient.

(2) On June 1, Carl's account balance was the same as its closing balance was for May, namely $2,600. Depending on whether Carl made a $120 deposit or a $50 withdrawal on

June 15, Carl's account balance on June 16 was either \$2,720 or \$2,550. It follows from the information given in (2) that Carl's balance on June 16 was \$2,550. Therefore, Carl began making \$50 withdrawals on or before June 15. These observations can be used to give at least two possible ranges. Carl could have had an account balance of 347 \$2,680 on January 1, made one \$120 deposit on January 15, and then made a \$50 withdrawal in each of the remaining 11 months of the year (this gives a closing balance of \$2,600 for May), which gives a range of monthly closing balances of (50)(11). Also, Carl could have had an account balance of \$2,510 on January 1, made \$120 deposits on January 15 and on February 15, and then made a \$50 withdrawal in each of the remaining 10 months of the year (this gives a closing balance of \$2,600 for May), which gives a range of monthly closing balances of (50)(10); NOT sufficient.

Given both (1) and (2), it follows from the remarks above that Carl began making \$50 withdrawals on June 15. Therefore, the changes to Carl's account balance for each month of last year are known. Since the closing balance for May is given, it follows that the closing balances for each month of last year are known, and hence the range of these 12 known values can be determined. Therefore, (1) and (2) together are sufficient! The correct answer is C; both statements together are sufficient.

3.

Before analyzing the statements, let's consider different scenarios for the range and the median of set A. Since we have an even number of integers in the set, the median of the set will be equal to the average of the two middle numbers. Further, note that integer 2 is the only even prime and it cannot be one of the two middle numbers, since it is the smallest of all primes. Therefore, both of the middle primes will be odd, their sum will be even, and their average (i.e. the median of the set) will be an integer. However, while we know that the median will be an integer, it is unknown whether this integer will be even or odd. For example, the average of 7 and 17 is 12 (even), while the average of 5 and 17 is 11 (odd). Next, let's consider the possible scenarios with the range. Remember that the range is the difference between the greatest and the smallest number in the set. Since we are dealing with prime numbers, the greatest prime in the set will always be odd, while the smallest one can be either odd or even (i.e. 2). If the smallest prime in the set is 2, then the range will be odd, otherwise, the range will be even. Now, let's consider these scenarios in light of each of the statements. (1) SUFFICIENT: If the smallest prime in the set is 5, the range of the set, i.e. the difference between two odd primes in this case, will be even. Since the median of the set will always be an integer, the product of the median and the range will always be even. (2) INSUFFICIENT: If the largest integer in the set is 101, the range of the set can be odd or even (for example, 101 – 3 = 98 or 101 – 2 = 99). The median of the set can also be odd or even, as we discussed. Therefore, the product of the median and the range can be either odd or even. The correct answer is A.

4.

(1) INSUFFICIENT: Statement (1) tells us that the range of S is less than 9. The range of a set is the positive difference between the smallest term and the largest term of the set. In this case, knowing that the range of set S is less than 9, we can answer only MAYBE to the question "Is $(x + y) < 18$". Consider the following two examples: Let $x = 7$ and $y = 7$. The range of S is less than 9 and $x + y < 18$, so we conclude YES. Let $x = 10$ and $y = 10$. The range of S is less than 9 and $x + y > 18$, so we conclude NO. Because this statement does not allow us to answer definitively Yes or No, it is insufficient. (2) SUFFICIENT: Statement (2) tells us that the average of x and y is less than the average of the set S. Writing this as an inequality: $(x + y)/2 < (7 + 8 + 9 + 12 + x + y)/6$; $(x + y)/2 < (36 + x + y)/6$;
$3(x + y) < 36 + (x + y)$; $2(x + y) < 36$; $x + y < 18$. Therefore, statement (2) is SUFFICIENT to determine whether $x + y < 18$.

5.

From Statement (1) alone, we can conclude that the range of the terms of S is either 3 or 7. (These are the prime numbers less than 11, excluding 2 and 5, which are both factors of 10.) Since the question states that the range of S is equal to the average of S, we know that the average of the terms in S must also be either 3 or 7. This alone is not sufficient to answer the question. From Statement (2) alone, we know that S is composed of exactly 5 different integers. this means that the smallest possible range of the terms in S is 4. (This would occur if the 5 different integers are consecutive.) This is not sufficient to answer the question.

From Statements (1) and (2) together, we know that the range of the terms in S must be 7. This means that the average of the terms in S is also 7. It may be tempting to conclude form this that the sum of the terms in S is equal to the average (7) multiplied by the number of terms (5) = 7 × 5 = 35. However, while Statement (2) says that S is composed of 5 different integers, this does not mean that S is composed of exactly 5 integers since each integer may occur in S more than once. Two contrasting examples help to illustrate this point: S could be the set {3, 6, 7, 9, 10}. Here, the range of S = the average of S = 7. Additionally, S is composed of 5 different integers and the sum of all the integers in S is 35. S could also be the set {3, 6, 7, 7, 9, 10}. Here, the range of S = the average of S = 7. Again, S is composed of 5 different integers. However, here the sum of S is 42 (since one of the integers, 7, appears twice.). The correct answer is E: Statements (1) and (2) TOGETHER are NOT sufficient.

6.

In a set consisting of an odd number of terms, the median is the number in the middle when the terms are arranged in ascending order. In a set consisting of an even number of terms, the median is the average of the two middle numbers. If S has an odd number of terms, we know that the median must be the middle number, and thus the median must be even (because it is a set of even integers). If S has an even number of terms, we know that the median must be the average of the two middle numbers, which are both even, and the

average of two consecutive even integers must be odd, and so therefore the median must be odd. The question can be rephrased: "Are there an even number of terms in the set?"
(1) SUFFICIENT: Let X_1 be the first term in the set and let its value equal x. Since S is a set of consecutive even integers, $X_2 = X_1 + 2$, $X_3 = X_1 + 4$, $X_4 = X_1 + 6$, and so on. Recall that the mean of a set of evenly spaced integers is simply the average of the first and last term. Construct a table as follows:

X_n	Value	Ave *n* Terms	Result	O or E
X_1	x	x	x	Even
X_2	$x + 2$	$\frac{2x + 2}{2}$	$x + 1$	Odd
X_3	$x + 4$	$\frac{3x + 6}{3}$	$x + 2$	Even
X_4	$x + 6$	$\frac{4x + 12}{4}$	$x + 3$	Odd
X_5	$x + 8$	$\frac{5x + 20}{5}$	$x + 4$	Even

Note that when there is an even number of terms, the mean is odd and when there are an odd number of terms, the mean is even. Hence, since (1) states that the mean is even, it follows that the number of terms must be odd. This is sufficient to answer the question (the answer is "no"). Note of caution: it doesn't matter whether the answer to the question is "yes" or "no"; it is only important to determine whether it is *possible* to answer the question given the information in the statement.
Alternatively, we can recognize that, in a set of consecutive numbers, the median is equal to the mean, and so the median must be even.
(2) INSUFFICIENT: Let X_1 be the first term in the set and let its value = x. The range of a set is defined as the difference between the largest value and the smallest value. Construct a table as follows:

Term	Value	Range *n* Terms	Div by 6?
X_1	x		
X_2	$x + 2$	2	No
X_3	$x + 4$	4	No
X_4	**$x + 6$**	**6**	**Yes**
X_5	$x + 8$	8	No
X_6	$x + 10$	10	No
X_7	**$x + 12$**	**12**	**Yes**

Note that if there are 4 terms in the set, the range of the set is divisible by 6, while if there are 7 terms in the set, the range of the set is still divisible by 6. Hence, it cannot be determined whether the number of terms in the set is even or odd based on whether the range of the set is divisible by 6. The correct answer is A.

7.

The median of a set of numbers is the middle number when the numbers are arranged in increasing order. For a set of 5 scores, the median is the 3rd score. We will call the set of scores $A = \{A_1, A_2, A_3, A_4, A_5\}$ and $B = \{B_1, B_2, B_3, B_4, B_5\}$ for Dr. Adams' and Dr. Brown's students, respectively, where the scores are arranged in increasing order within each set. Rephrasing the question using this notation yields "Is $A_3 > B_3$?" (1) INSUFFICIENT: This statement tells us only the highest and lowest score for each set of students, but the only thing we know about the scores in between is that they are somewhere in that range. Since the median is one of the scores in between, this uncertainty means that the statement is insufficient.
To illustrate, A_3 could be greater than B_3, making the answer to the question "yes": $A = \{40, 50, \mathbf{60}, 70, 80\}$
$B = \{50, 55, \mathbf{55}, 80, 90\}$ However, A_3 could be less than or equal to B_3, making the answer to the question "no":
$A = \{40, 50, \mathbf{60}, 70, 80\}$ $B = \{50, 60, \mathbf{70}, 80, 90\}$ (2) SUFFICIENT: This statement tells us that for every student pair, the B student scored higher than the A student, or $B_n > A_n$. This statement can be considered qualitatively. Every student in set B scored higher than *at least one* student in set A. The students in set B not only scored higher individually, but also as a group, so one can reason that the median score for set B is higher than the median score for set A. Therefore, $B_3 > A_3$, and the answer to the question is "no." But let's prove conclusively that the answer cannot be "yes." Constrain A_3 to be greater than B_3, then try to pair the students according to the restriction that $B_n > A_n$. For example, pick any number x between 0 and 100, and let's say that $A_3 > x$, or high (H), and that $B_3 < x$, or low (L). Since the scores are increasing order, the 1st and 2nd scores must be less than or equal to the 3rd, while the 4th and 5th scores must be greater than or equal to the 3rd. Thus we know whether all the other scores are high or low. $A = \{A_1, A_2, \boldsymbol{H}, A_4, A_5\} = \{L, L, \boldsymbol{H}, H, H\}$
$B = \{B_1, B_2, \boldsymbol{L}, B_4, B_5\} = \{L, L, \boldsymbol{L}, H, H\}$
In order to meet the restriction that $B_n > A_n$, each of the 3 high scorers (H) in set A must be paired with a high(er) scorer, but there are only 2 high scorers (H) in set B—not enough to go around! Conversely, the 3 low scorers (L) in set B *cannot* be paired with a high scorer (H) from set A, leaving only 2 potential study partners for them from set A—not enough to go around! There is no way for A_3 to be greater than B_3 and still meet the restriction that $B_n > A_n$, so $A_3 < B_3$. Thus, the answer can never be "yes," it is always "no," and this statement is sufficient. The correct answer is B.

8.

In order to determine the median of a set of integers, we need to find the "middle" value.
(1) SUFFICIENT: Statement one tells us that average of the set of integers from 1 to x inclusive is 11. Since this is a set of consecutive integers, the "average" term is always the exact middle of the set. Thus, in order to have an average of 11, the set must be the integers from 1 to 21 inclusive. The middle or median term is also is 11.
(2) SUFFICIENT: Statement two states that the range of the set of integers from 1 to x

inclusive is 20. In order for the range of integers to be 20, the set must be the integers from 1 to 21 inclusive. The median term in this set is 11. The correct answer is D.

9.
Range before transaction:
112-45=67
Range after transaction:
(94+24)-(56-20)=118-36=82
The difference is: 82-67=15
Answer is D

10.

Range: the difference between the greatest measurement and the smallest measurement.
In the question, combine 1 and 2, we still cannot know the value of q, then, we cannot determine which number of is the greatest measurement.
Answer is E

11.

Prior to median 25, there are 7 numbers.
To make the greatest number as greater as possible, these 7 numbers should cost the range as little as possible. They will be, 24, 23, 22, 21, 20, 19, 18.
So, the greatest value that can fulfill the range is: 18+25=43

12.

Answer is A.

A neat little trick to remember is that for any series that is an Arithmetic progression, namely difference between each successive term is constant, the median is always = mean. I'll try to prove it below. Let us say there are n terms. There are two pssibilities n is odd or n is even. Let us say the constant difference is d (2 in the case of this problem).

1st term: a, 2nd term = a+d, 3rd term = a+2*d .... nth term = a+(n-1)*d
Adding all you get Sum = a*n +[(1+2+3...+(n-1))]*d = a*n +[n*(n-1)/2]*d (another interesting result sum of the first n-1 integers is n*(n-1)/2]. Hence the Average = Sum/n = a+(n-1)/2*d

n is odd: Median = (n+1)/2 th term = a+(n-1)/2*d = Average so if n is odd we have proved avg always equal to mean.
n is even: Median = average of n/2 and (n/2+1)th term =[a+(n/2-1)*d+a+(n/2)*d]/2 = a+(n-1)*d/2 = Average so if n is even too we have proved avg always equal to mean. Thus (1) is always sufficient to answer such questions of if avg = median.

(2) Since only range is given we cant determine anything about the numbers in between so this informatino is insufficient. Hence answer is A.

Yeah. As for (1) - in a little simpler terms - if the numbers in the set are EQUALLY SPACED apart (this is the meaning of 'arithmetic progression'), then the median and the mean will be equal. In fact, in any set where there is left-right SYMMETRY if the numbers are plotted on a number line, the mean and the median will be the same.

13.

both unknowns are 'G', so solve for 'G':
G = L + range

--

may:
range = 15
L = 4.5
G = 15 + 4.5 = 19.5

--

june:

range = 16.5
L = 6.1
G = 16.5 + 6.1 = 22.6

--

overall 'L' = 4.5 (lower of the two 'L's)
overall 'G' = 22.6 (higher of the two 'G's)
overall range = 22.6 - 4.5 = 18.1

OR

The correct answer D, $18,100
First of all let us find the lowest price of the car sold in the two months period
ie may and june
since the least price of car sold in may is 4500 less than the least cost of the car sold in june which was 6100

so we have the lowest value in order to find the range=4500

consider the month of may range is given as 15,000

so 15000=max cost of car-least cost of a car
so 15000=max cost of car-4500
so max cost of car in may=19500
similarly for june max cost of car=22600

so now we have the highest price and the lowest price
so range=22600-4500=18100

14.

Sum = 55 * 5 = 275

x + x + 55 + 55 + 3x + 20 = 275
x = 29, 3x + 20 = 107, range = 78

OR

Let's choose the numbers as x, y, 55, z, 3x+20

So range here is 3x + 20 - x = 2x + 20 ------------- (1)

So if the range has to be maximum, than x has to be maximum..

We also know that the average is 55.. that means

x + y + 55 + z + 3x + 20 = 275

or 4x + y + z = 200 ------------- (2)

Now when would x be maximum, only when y and z are minimum.

So
1. What is the minimum value of z? Well, it has to be 55 coz the median is 55, so z cannot be less than 55
2. What is the minimum value of y? Well it has to be equal to x. It can't be less than x..

So with the above deductions, equation (2) becomes

4x + x + 55 = 200

or 5x = 145

or x = 29

Putting this value in (1)

we get 2*(29) + 20 = 78

So answer should be A i.e. 78

Standard Deviation

1.

In determining the standard deviation, the difference between each measurement and the mean is squared, and then the squared differences are added and divided by the number of measurements. The quotient is the variance and the positive square root of the variance is the standard deviation.
(1) If the variance is 4, then the standard deviation, which is less than 3; SUFFICIENT.
(2) For each measurement, the difference between the mean and that measurement is 2. Therefore, the square of each difference is 4, and the sum of all the squares is 4 × 20 = 80. The standard deviation is $\sqrt{(80/20)} = \sqrt{4} = 2$, which is less than 3; SUFFICIENT. The correct answer is D; each statement alone is sufficient.

2.

The data set with the least standard deviation will be the data set with elements most closely clustered around the mean of the data set, and the data set with the greatest standard deviation will be the data set with elements that are spread out farthest from the mean of the data set. Because set I is symmetric about 74 (73 is 1 less than 74 and 75 is 1 more than 74; 72 is 2 less than 74 and 76 is 2 more than 74), the mean of set I is 74. Because every number in set II is 74, the mean of set II is 74. The mean of set III is 74.6. The elements of set II do not deviate at all from 74, so set II has the least standard deviation. The most that any element of set I differs from 74 is 2, but there are elements of set III that differ from 74.6 by 12.6 and 14.4. Therefore, set III has a greater standard deviation than set I, which has a greater standard deviation than set II. The correct answer is D.

3.

Note that if all the values in a data set are equal to the same number, say x, then the average of the data set is x, the difference between each data value and the average is x – x = 0, the sum of the squares of these differences is 0, and so the standard deviation is 0. On the other hand, if the values in a data set are not all equal to the same number, then the standard deviation will be positive.
(1) If each of the 10 nests had 4 eggs, then the average would be 4 and the standard deviation would be 0. If 8 nests had 4 eggs, 1 nest had 3 eggs, and 1 nest had 5 eggs, then the average would be 4 and the standard deviation would be positive; NOT sufficient.

(2) Since all of the data values are equal to the same number, the standard deviation is 0; SUFFICIENT. The correct answer is B; statement 2 alone is sufficient.

4.

If 5 were added to each score, the mean would go up by 5, as would the median. However, the spread of the values would remain the same, simply centered around a new value. So, the standard deviation would NOT change. The correct answer is D.

5. E
6. D
7. E
8. E
9. C
10. 65 to 85
11. 30
12. |a| S
13. |a/b| × (S)

14.

Standard deviation is a measure of how far the data points in a set fall from the mean. For example, {5, 5, 6, 7, 7} has a small standard deviation relative to {1, 4, 6, 7, 10}. The values in the second set are much further from the mean than the values in the first set. In general, a value that drastically increases the range of a set will also have a large impact on the standard deviation. In this case, 14 creates the largest spread of the five answer choices, and will therefore be the value that most increases the standard deviation of Set *T*. The correct answer is E.

15.

The procedure for finding the standard deviation for a set is as follows: 1) Find the difference between each term in the set and the mean of the set. 2) Average the squared "differences." 3) Take the square root of that average. Notice that the standard deviation hinges on step 1: finding **the difference between each term in the set and the mean of the set**. Once this is done, the remaining steps are just calculations based on these "differences." Thus, we can rephrase the question as follows: "What is the difference between each term in the set and the mean of the set?" (1) SUFFICIENT: From the question, we know that Q is a set of consecutive integers. Statement 1 tells us that there are 21 terms in the set. Since, in any consecutive set with an odd number of terms, the middle value is the mean of the set, we can represent the set as 10 terms on either side of the middle term x: [$x - 10$, $x - 9$, $x - 8$, $x - 7$, $x - 6$, $x - 5$, $x - 4$, $x - 3$, $x - 2$, $x - 1$, $\boldsymbol{x}$, $x + 1$, $x + 2$, $x + 3$, $x + 4$, $x + 5$, $x + 6$, $x + 7$, $x + 8$, $x + 9$, $x + 10$] . Notice that the difference between the mean (x) and the first term in the set ($x - 10$) is 10. The difference between the mean (x) and the second term in the set ($x - 9$) is 9. As you can see, we can actually find the difference between each term in the set and the mean of the set without knowing the specific value of each term in the set! (The only reason we are able to do this is because we know that the set abides by a specified consecutive pattern and because we are told the number of terms in this set.) Since we are able to find the "differences," we can use these to calculate the standard deviation of the set. Although you do not need to do this, here is the actual calculation: Sum of the squared differences: $10^2 + 9^2 + 8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2 + (-1)^2 + (-2)^2(-3)^2 + (-4)^2 + (-5)^2 + (-6)^2(-7)^2 + (-8)^2 + (-9)^2 + (-10)^2 = 770$.

Average of the sum of the squared differences: $\frac{770}{21} = 36\frac{2}{3}$

The square root of this average is the standard deviation: $\sqrt{36\frac{2}{3}} \approx 6.06$

(2) NOT SUFFICIENT: Since the set is consecutive, we know that the median is equal to the mean. Thus, we know that the mean is 20. However, we do not know how big the set is so we cannot identify the difference between each term and the mean. Therefore, the correct answer is A.

16.

(1) SUFFICIENT: The average of data set $B = \{1, 2, 3\}$ is 2. So in data set $A = \{1, 2, x\}$ as x increases above 3, it gets further and further from the average. This necessarily increases its standard deviation, so A necessarily has a greater standard deviation than B.

(2) INSUFFICIENT: Statement (2) is insufficient since there are two different values of x less than 1 that give different answers to the question. For example, let $x = 0$ so $A = \{0, 1, 2\}$. Then A has the same standard deviation as B.

Now let $x = -100$, so $A = \{-100, 1, 2\}$. Clearly A has a larger standard deviation than B since its data is much more spread out. Since we have found two different values of x that give different answers to the question, statement (2) is insufficient.

The correct answer is A.

17.

10-0.3=9.7 10+0.3=10.3 the number within 1 standard deviation should be between 9.7-10.3 so there are six numbers within 1 standard deviation 6/8=75%

Answer is D

18.

Average=100

1 standard deviation below the mean: less than 100-22.4=77.6

Obviously, 70 and 75 can fulfill the requirements.

Answer is B
19.
1 and 2 standard deviations below the mean=>number of the hours at most is 21-6=15, at least is 21-2*6=9.
Answer is D
20.
Mean 8.1
Standard deviation 0.3
Within 1.5 standard deviations of the mean=[8.1-0.3*1.5,8.1+0.3*1.5]=[7.65,8.55]
All the numbers except 7.51 fall within such interval
Answer is 11
21.
d^2=[(a1-a)^2+(a2-a)^2+....+(an-a)^2]/n
When we added 6 and 6, the numerator remained unchanged but the denominator increased, so, the new deviation is less than d.
Answer is E
22.
For statement 1, we know that 68% are within [m-d, m+d], so, the percent greater than m+d will be (1-0.68)/2.
For statement 2, we know that 16% is less than m-d, considering the distribution is symmetric about the mean m, we can get, 16% is greater than m+d.
Answer is D

23.

* if you **ADD OR SUBTRACT A CONSTANT** to/from all the values in a set, then the **standard deviation will remain exactly the same**.
visual equivalent: imagine sliding all the data points along a number line, by exactly the same amount, to the right or left. if you do this, then the average spread obviously won't change, because, in fact, none of the spreads anywhere in the set changes at all.
this is where the referenced post went wrong: even if you have a set in which the values are wildly different from one another, the standard deviation will not change if a constant is added to or subtracted from all the values.

* if you **INCREASE OR DECREASE ALL THE VALUES BY A FIXED FACTOR / PERCENTAGE**, then the **standard deviation will increase or decrease by the same percentage**.
visual equivalent: imagine drawing a number line with the set on an elastic band, and stretching or contracting the elastic band to mimic the % increase / decrease applied to the set. if you do this, then the average spread must increase / decrease by the same %, because all the spreads will increase / decrease by that %.

Make sure you know that, when ALL numbers in a set are multiplied or divided by some number, the mean and standard deviation are multiplied /divided by the same number.

This includes increasing or decreasing all the numbers in the set by some percentage (which can be accomplished by multiplication: e.g., 30% increase = multiplication by 1.3).

Using this principle, statement (1) tells us that both the mean and the standard deviation of the set will decrease by 30%. Therefore, the new standard deviation will decrease to 7 gallons. SUFFICIENT.

Statement (2) tells us nothing about standard deviation, which measures SPREAD of numbers. If we achieved the 63 gallons by taking most of the water out of the tanks that were already lowest, then the standard deviation will be huge (because you'll have some tanks almost full and some almost empty). If we got there by taking most of the water out of the fullest tanks, then the standard deviation will be a lot smaller. INSUFFICIENT.

24.

Standard deviation = deviation from the mean.

from the given data 0,2,4,6,8... mean is 4

So for the least deviation from mean 4, if we choose both the number as 4, the average will not change eventually S.D will not change also

Solutions to Quant Session 2 – 75 questions on Numbers, Inequalities, and Mods

1.

OA: A

the word "parity", as used in the following discussion, means "whether something is even or odd", in the same way in which "sign" means whether a number is positive or negative.

the best way to approach things like this is to **break down the compound statements into statements about the parity of the individual variables**.
to do that, you'll almost certainly have to **split the statements into cases**.

"xy + z is odd"
two numbers can add to give an odd sum only if they have opposite parity. hence:
case 1: xy is odd, z is even
there's only one way this can happen:
x = odd, y = odd, z = even. **(1)**
case 2: xy is even, z is odd
there are 3 ways in which this can happen:
x = even, y = even, z = odd (2a)
x = odd, y = even, z = odd (2b)
x = even, y = odd, z = odd (2c)

this is a bit awkward, but, once you've divided the question prompt up into cases, all you have to do is look at your results, check the cases, and you'll have an answer.

—

statement (1)
the easiest way to handle expressions like this is to factor out common terms. you can handle the statement without doing so, but it's more work that way.
pull out x:
x(y + z) is even.
this means that *at least one* of x and (y + z) is even.
* if x is even, regardless of the parity of (y + z), then the answer to the prompt question is "yes" and we're done.
* the other possibility would be x = odd and (y + z) = even. this is impossible, though, as it doesn't satisfy any of the cases above.
therefore, the answer must be "yes".
sufficient.

—

statement (2)
this means that y and xz have opposite parity.
* y = even, xz = odd —> this means x = odd, y = even, z = odd. that's case (2b), which gives "no" to the question.
at this point you're done, because STATEMENTS CAN'T CONTRADICT EACH OTHER, se you know that "yes" MUST be a possibility with this statement (as statement #1 gives exclusively "yes" answers).
if you use this statement first, you'll have to keep going through the cases.
insufficient.

ans = a

2.
this problem involves two fractions that are added together. for no other reason than that 'it's the normal thing to do with two fractions added together', let's find the common denominator:
w/x + y/z = wz/xz + xy/xz = (wz + xy)/xz

therefore
the question can be rearranged to:
is (wz + xy)/xz – which is the same thing as w/x + y/z – odd?

— (2) alone —

if wz + xy is an odd integer, then all of its factors are odd. this means that (wz + xy)/xz, which is guaranteed to be an integer**, must also be odd – because it's a factor of an odd number.

sufficient

**we know this is an integer because it's equal to w/x + y/z, which, according to the information given in the problem statement, is integer + integer.

— (1) alone —

try to come up with contradictory examples**:

w=2, x=1, y=3, z=1 (so that wx + yz = 5 = odd, per the requirement):
w/x + y/z = 2 + 3 = 5 = odd

w=2, x=2, y=3, z=1 (so that wx + yz = 7 = odd, per the requirement):
w/x + y/z = 1 + 3 = 4 = even

insufficient

**of course, if you're at a loss for the theory, you should try this for statement (1) too ... but you'll find that all the examples you get are odd.

—

answer = b

3.

JUST PLUG IN NUMBERS.

statement (1)

let's just PICK A WHOLE BUNCH OF NUMBERS WHOSE GCF IS 2 and watch what happens. let's try to make the numbers diverse.
say,
4 and 6
6 and 8
8 and 10
10 and 12
...
4 and 10
6 and 14
6 and 16
8 and 18
8 and 22

...
in all nine of these examples, the remainders are greater than 1. in fact, there is an obvious pattern, which is that **they're all even**, since the numbers in question must be even.

in fact, i just thought of this, which is a much nicer, more ground-level approach to statement one:
in statement 1, both m and p are even. therefore, the remainder is even, so it's greater than 1.

done.

sufficient.

--

statement (2)
just pick various numbers whose lcm is 30.
notice the numbers selected above:
5 and 6 --> remainder = 1
10 and 15 --> remainder = 5 > 1
insufficient.

ans (a)

4.

Answer: B

(1)
this is a disguised way of saying 'n is prime'
therefore, insufficient

(2)
this says any two factors. that means **any** two factors – i.e., ALL pairs of factors have an odd difference.
there's only one way to do this: one odd factor and one even factor. (as soon as you get 2 odd factors or 2 even factors, you get an even difference by subtracting them.)
2 is the only # with only 1 odd factor and only 1 even factor.
therefore, sufficient

5.
Take a prime number and figure out a specific soln for that prime number.
Let p = 5. So, excluding 1, the other numbers that have no factors common with 5 are 2,3,4.
Let p = 7. So, excluding 1, the other numbers that have no factors common with 7 are 2,3,4,5,6
Do you see the pattern? For any prime number, all the numbers less than it will have no factors

in common with it except 1.
So f(p) = p – 2 Answer is B. **NOOOOOOOOOO**

We need to include 1 and hence the correct answer is p–1 (A).

Pick a prime number for p. Let's say p=5.

The positive integers less than 5 are 4, 3, 2, and 1.

5 and 4 share only 1 as a factor
5 and 3 share only 1 as a factor
5 and 2 share only 1 as a factor
5 and 1 share only 1 as a factor

There are four positive integers, therefore, that are both less than 5 and share only 1 as a factor. In other words, we include 1 in this set of integers.

6.

Let's first consider the prime factors of h(100). According to the given function,
h(100) = 2*4*6*8*...*100

By factoring a 2 from each term of our function, h(100) can be rewritten as
2^50*(1*2*3*...*50).

Thus, all integers up to 50 - **including all prime numbers up to 50** - are factors of h(100).

Therefore, h(100) **+ 1** cannot have any prime factors 50 or below, since dividing this value by any of these prime numbers will yield a remainder of 1.

Since the smallest prime number that can be a factor of h(100) + 1 has to be greater than 50, The correct answer is E.

7.
n is an integer
is n odd?
yes/no question, so I will try to prove it wrong (that is, get a yes and a no based upon the statements)
(1) n/3
n could be 6 (that is divisible by 3). Is n odd? No
n could be 9 (that is divisible by 3). Is n odd? Yes
Elim A and D
(2) 2n has twice as many factors as n
n could be 1, which has one factor; 2n would be 2, which has two factors; is n odd? Yes

n could be 2, which has two factors; 2n would be 4, which has three factors. Oops, can't use this combo of numbers (has to make statement 2 true, and this combo doesn't)

What's going on here?
general rule: 2n will be divisible by 2 and also by whatever number 2n is.
If I make n an even number, even numbers are already divisible by 2. So 2n will only be divisible by one new number, equal to 2n. That is, I add only one new factor for 2n. **[editor: there's a mistake in this explanation - see below for a correction]**
Any even number, by definition, has at least two factors - 1 and 2. So I would need to add at least two more factors to double the number of factors. But I can't - the setup of statement 2 only allows me to add one new factor if n is even. So I can never make statement 2 true using an even number for n.

Sufficient. Answer is B.
let's say a number has "n" different factors.
when you multiply this number by 2, you POTENTIALLY create "n" MORE factors - by doubling each factor.
HOWEVER,
the only way that ALL of these factors can be NEW (i.e., not already listed in the original n factors) is if they are ALL ODD.
if there are ANY even factors to start with, then those factors will be repeated in the original list. (for instance, note that 2, 4, 26, and 52 all appear in both lists above.) therefore, **if the number is even, then the number of factors will be less than doubled** because of the repeat factors.

thus if statement (2) is true, then the number must be odd.

8.
Start with statement 2. This doesn't tell us one value of d, so elim. B and D.

Statement 1: 10^d is a factor of f. This isn't going to be sufficient. If you're not sure why try the easiest possible positive integers. Is 10^1 = 10 a factor of f? Yes, so 1 is a possible value for d. Is 10^2 = 100 a possible factor of f? Yes, so 2 is a possible value for d. I just found 2 possible values for d. Elim. A. Only C and E are left.

d is a pos int (given in stem) and is greater than 6 (statement 2). Smallest possibility, then, is 7. If d is anything greater than 7, then 7 will work too (eg, if d actually is 8, then 7 would also satisfy both statements and we wouldn't be able to tell, just from the statements, whether d is 7 or 8). So it's either 7, exactly, which is sufficient, or it's something greater than 7, which is not sufficient.

So how many 10's are in f?
write down the numbers that contain 2s and 5s (only those)
30*28*26*25*24*22*20*18*16*15*14*12*10*8*6*5*4*2

Now ask yourself Is my limiting factor going to be 5 or is it going to be 2?

It's going to be 5 because there are many more 2's up there. So circle the numbers that contain 5's:
30, 25, 20, 15, 10, 5
How many 5's do you have? Seven 5's (don't forget – 25 has two 5's!), so you can make seven 10's. That's it. Answer is C.

"limiting factor" means "which is least common or likely." Think of it this way: there are many more multiples of 2 than there are multiples of 5. In probability terms, a number is more likely to be even than to be a multiple of 5. In divisibility terms, take some large number that is divisible by both 2 and 5, and it is likely to have more factors of 2 than 5.

For example: 400 = 4*10*10 = (2*2)(2*5)(2*5) = (2^4)(5^2).

I know, numbers with more factors of 5 than factors of 2 exist...this is just a bet we make to ease the computation.

In general, the larger the factor, the less likely it is to divide evenly into a number. The larger the factor, the more of a "limiting factor" it is.

here's all you have to do:
forget entirely about 10, 20, and 30, and **ONLY THINK ABOUT PRIME FACTORIZATIONS**.
(TAKEAWAY: this is the way to go in general – when you break something down into primes, you should not think in hybrid terms like this. instead, just translate *everything* into the language of primes.)

each PAIR OF A '5' AND A '2' in the prime factorization translates into a '10'.

there are **seven 5's**: one each from 5, 10, 15, 20, and 30, and two from 25.

there are waaaaaaayyyyy more than seven 2's.

therefore, **30! can accommodate as many as seven 10's** before you run out of fives.

—

statement 2 is clearly insufficient.

statement 1, by itself, means that d can be anything from 1 to 7 inclusive.

together, d must be 7.

ans (c)

9.

96 =

6*8*2 or 2*8*6 or 826 or 628.....
3*8*4 or 483 or 843 or ....
According to 1: the number is odd; We have only one odd digit: 3. – correct
while II says: hundreds digit of m is 8; There are many combination: 682 or 483 or.... incorrect.

96 = 2*2*2*2*2*3,

statement 1: m is odd, so unit's digit could be 1,3,5,7,9.
But we have only one odd factor in 96(product of digits of m) i.e. 3. Therefore, unit's digit of m is 3. – sufficient

statement 2: hundred's digit is 8, so we are left with 2*2*3. Therefore, m could be 826, 843, 834, 862. So no unique unit's digit. Insufficient

10.
the correct answer: B

if we are told that four different prime numbers are factors of 2n then can't i further assume that one of those four prime numbers is 2 (since it's 2n)

it's possible that 2 is already a factor of n to start with, in which case n itself would still have 4 different prime factors (because, in that case, the additional 2 would not change the total number of prime factors).

for instance, if n = 3x5x7 = 105 (which has three prime factors), then 2n = 2x3x5x7 = 210 has four prime factors.
if n = 2x3x5x7 = 210, which has four prime factors, then 2n = 2x2x3x5x7 = 420, which still has two prime factors.
therefore, #1 is not sufficient.

11.
the question is asking whether k has a factor that is greater than 1, but less than itself.
if you're good at these number property rephrasings, then you can realize that this question is equivalent to "is k non–prime?", which, in turn, because it's a data sufficiency problem (and therefore we don't care whether the answer is "yes" or "no", as long as there's an answer), is equivalent to "is k prime?".
but let's stick to the first question – "does k have a factor that's between 1 and k itself?" – because that's easier to interpret, and, ironically, is easier to think about (on this particular problem) than the prime issue.

—

key realization:

every one of the numbers 2, 3, 4, 5, ..., 12, 13 is a factor of 13!.

this should be clear when you think about the definition of a factorial: it's just the product of all the integers from 1 through 13. because all of those numbers are in the product, they're all factors (some of them several times over).

—

consider the lowest number allowed by statement 2: 13! + 2.
note that 2 goes into 13! (as shown above), and 2 also goes into 2. therefore, 2 is a factor of this sum (answer to question prompt = "yes").

consider the next number allowed by statement 2: 13! + 3.
note that 3 goes into 13! (as shown above), and 3 also goes into 3. therefore, 3 is a factor of this sum (answer to question prompt = "yes").

etc.
all the way to 13! + 13.
works the same way each time.
so the answer is "yes" every time —> sufficient.

—

in this problem, the prompt asks, "**Is there a factor p** such that...?"
this means that, *if you can show that there is even one such factor*, then it's "sufficient" and you are DONE.
we have ascertained that every one of the "k"s in that range has *at least one such factor*.
to wit, 13! + 2 has the factor 2; 13! + 3 has the factor 3; ...; 13! + 13 has the factor 13.
that's all we need to know.
sufficient.

you are right that it's difficult to ascertain whether numbers greater than 13 are factors of these "k"s. luckily, we don't have to care about that.

12.

OA: D

Since it has only 2 prime factors but 6 factors (4 of which are 1, 3, 7, k) this means that the prime factors must be combined to generate the other 2 factors – the other 2 can only be either 3 which means 3x3=9 and 3.7=21 is a factor OR 7 which means the other 2 factors are 21 and 49.

SHORTCUT METHOD:
if you know the following useful fact, then you can solve this problem much more quickly.
USEFUL FACT: if a, b, ... are the EXPONENTS in the prime factorization of a number,

then the total number of factors of that number is the product of (a + 1), (b + 1), ...
example:
540 = (2^2)(3^3)(5^1), in which the exponents are 2, 3, and 1. therefore, 540 has (2 + 1)(3 + 1)(1 + 1) = 3 x 4 x 2 = 24 different factors.

with this shortcut method, realize that 6 (the total number of factors) is 3 x 2. therefore, the exponents in the prime factorization must be 2 and 1, in some order.
therefore, **there are only two possibilities: k = (3^2)(7^1) = 63, or k = (3^1)(7^2) = 147.**

statement (1) includes 63 but rules out 149, so, sufficient.
statement (2) includes 63 but rules out 149, so, sufficient.
answer = (d).

—

IF YOU DON'T KNOW THE SHORTCUT:

statement (1)
if 3^2 is a factor of k, then so is 3^1.
therefore, we already have four factors: 1, 3^1, 3^2, and 7.
but we also know that (3^1)(7) and (3^2)(7) must be factors, since 3^2 and 7 are both part of the prime factorization of k.
that's already six factors, so we're done: k must be (3^2)(7). if it were any bigger, then there would be more than these six factors.
sufficient.

statement (2)
if 7 is a factor of k, but 7^2 isn't, then the prime factorization of k contains EXACTLY one 7.
therefore, we need to find out how many 3's will produce six factors when paired with exactly one 7.
in fact, it's data sufficiency, so we don't even have to find this number; all we have to do is realize that adding more 3's will always increase the number of factors, so, there must be exactly one number of 3's that will produce the correct number of factors. (as already noted above, that's two 3's, or 3^2.)
sufficient.

13.
when you take the product of two numbers, all you're doing, in terms of primes, is throwing all the prime factors of both numbers together into one big pool.
therefore, the original question – 'what's the greatest prime factor of the product?' – can be rephrased as,
what's the greatest prime that's a factor of either t or n?

(1)
because the gcf only tells us which primes are in BOTH t and n. there could be great big fat

primes that are factors of only one of them, and they wouldn't show up in the gcf.
insufficient.

(2)
the lcm of two numbers contains EVERY prime that appears in either one of the two numbers (because it's a multiple of both numbers). therefore, whatever is the largest prime factor of the lcm is also the largest prime that goes evenly into either t or n.
sufficient.

—

if you don't realize why the relationships between lcm/gcf and primes, stated above, are what they are, you can just try a few cases and watch the results for yourself. for instance, consider the two numbers 30 (= 2 x 3 x 5) and 70 (= 2 x 5 x 7).
the gcf of these 2 numbers is 10 (= 2 x 5), which doesn't show anything about the presence of the prime factor 7 in one of the numbers.
the lcm of these 2 numbers is 210 (= 2 x 3 x 5 x 7), which contains all of the primes found in either number.
Ans. B

OR
(1) The first statement does not tell us about factors that are not common to n and t. One of those factors might be greater or less than 5. Tis statement alone is NOT SUFFICIENT

(2) The LCM is 105 which can be factored as 3*5*7. Since LCM incorporates all the factors of n and t we know the greatest factor for the two numbers is 7. Hence this statement is sufficient.

OR

consider the 'prime box' approach (= an imaginary box that contains all the numbers in the prime factorization of a number, for those of you who are uninitiated into our curriculum).

you're looking for the greatest prime # that would be in the 'box' obtained from dumping all the factors of n and all the factors of t, including all repetitions, into a bigger box. (this is what multiplication does: it multiplies the complete factorization of one number by that of another. for instance, 12 = 2x2x3 and 20 = 2x2x5, so 12 x 20 = 2x2x2x2x3x5.) therefore, the question can be rephrased as follows: what is the greatest prime # that is a factor **of either** t or n ?

(1) this only tells is that the greatest number that is in **both** factorizations – those of n and t – is 5. but there could be a larger factor that is part of only one of the factorizations. for instance:
– it's possible that n = t = 5. then the greatest prime factor of nt is 5.
– it's possible that n = 5 and t = 35. then the greatest prime factor of nt is 7.
insufficient.

(2) the least common multiple contains every factor of t or n at least once. (it has to; if, say, t had a factor that wasn't contained in it, then it would fail to be a multiple of t.) so, the biggest prime

factor of this # will also be the biggest prime factor of the product nt.
sufficient.
try a few combinations of n and t if you aren't convinced.

answer = b

14.
Answer is A.

From (1), I could figure (t+3)(t+2) will always have a remainder 2, hence SUFFICIENT

I had trouble with (2) as I could not come up with an algebraic approach. I understand I can plug numbers to see that t^2 = 36 and t^2 = 64 fit the criterion BUT yield different remainders when the corresponding values of t are plugged into t^2 + 5t +6 and hence INSUFFICIENT.

OR

one fact that's pretty cool, and which happens to apply to this problem, is that *you can do normal arithmetic with remainders, as long as all the remainders come from division by the same number*. the only difference is that, if/when you get numbers that are too big to be authentic remainders (i.e., they're equal to or greater than the number you're dividing by), you have to take out as many multiples of the divisor as necessary to convert them back into "legitimate" remainders again. you can think of the remainders as on an odometer that rolls back to 0 whenever you reach the number you're dividing by.

so with statement (1), all the remainders are upon division by 7, so we can do normal arithmetic with them:
if t gives a remainder of 6, then t^2 = t x t gives a remainder of 6 x 6 = 36 —> this is more than 7, so we take out as many 7's as possible: 36 – 35 = 1.
if t gives a remainder of 6, then 5t gives a remainder of 5(6) = 30 —> this is more than 7, so we take out as many 7's as possible: 30 – 28 = 2.
and finally, 6 itself gives a remainder of 6.
therefore, the grand remainder when t^2 + 5t + 6 is divided by 7 should be 1 + 2 + 6 = 9 —> take out one more seven —> remainder will be 2.
sufficient.

by the way, much more generally (and therefore perhaps more importantly), **the patterns in remainder problems will always emerge fairly early when you plug in numbers.** therefore, if you don't IMMEDIATELY realize a good theoretical way to do a remainder problem, you should get on the number plugging RIGHT AWAY.

with statement (1), generate the first 3 numbers for which the statement is true: 6, 13, 20.
try 6: 36 + 30 + 6 = 72, which yields a remainder of 2 upon division by 7.
try 13: 169 + 65 + 6 = 240, which yields a remainder of 2 upon division by 7.
try 20: 400 + 100 + 6 = 506, which yields a remainder of 2 upon division by 7.
i'm convinced. (again, remember that PATTERNS EMERGE EARLY in remainder problems. 3

examples may not be enough for other types of pattern recognition, but that's usually pretty good in a remainder problem.)

with statement (2), as a poster has already mentioned above, find the first two t^2's that actually do this, which are 1^2 = 1 and 6^2 = 36.
if t = 1, then 1 + 5 + 6 = 12, which yields a remainder of 5 upon division by 7.
if t = 6, then 36 + 30 + 6 = 72, which yields a remainder of 2 upon division by 7.
insufficient.

OR
remainder problems usually show patterns after a very, very small number of plug–ins.

statement (1):
it's easy to generate t's that do this: 6, 13, 20, 27, ... (note that 6 is a member of this list, and an awfully valuable one at that; it's quite easy to plug in)

try 6: 36 + 30 + 6 = 72; divide by 7 —> remainder 2
try 13: 169 + 65 + 6 = 240; divide by 7 —> remainder 2
try 20: 400 + 100 + 6 = 506; divide by 7 —> remainder 2

by this point i'd be convinced.
note that 3 plug–ins is NOT good enough for a great many problems, esp. number properties problems. however, as i said above, remainder problems don't keep secrets for long.

sufficient.

statement (2):
it's harder to find t's that do this. however, **the gmat is nice to you. if examples are harder to find, then the results will usually come VERY quickly once you find those examples.**
just take perfect squares, examine them, and see whether they give the requisite remainder upon division by 7.

the first two perfect squares that do so are 1^2 = 1 and 6^2 = 36.
if you don't recognize that 1 ÷ 7 gives remainder 1, then you'll have to dig up 6^2 = 36 and 8^2 = 64. that's not that much more work.

in any case, you'll have
1 + 5 + 6 = 12 —> divide by 7; remainder = 5
36 + 30 + 6 = 72 —> divide by 7, remainder = 2 (the work for this was already done above; you should NOT do it twice. i'm reproducing it here only for the sake of quick understanding.)

or
36 + 30 + 6 = 72 —> divide by 7, remainder = 2 (the work for this was already done above; you should NOT do it twice. i'm reproducing it here only for the sake of quick understanding.)
64 +40 + 6 = 110 —> divide by 7, remainder = 5

either way, insufficient within the first two plug–ins!

answer (a)

15.
A) (8*n + 5)/4 –> sufficient

B) p is odd. So in the sum one is odd and one is even.

p = e^2 + o^2

e^2 will be divisible by 4.
o^2 = (2n+1)(2n+1) / 4 = (4n^2 + 4n + 1)/4 –> sufficient

you can always plug in a bunch of numbers until you've satisfied yourself that the statements are sufficient.
for (1), just find the first few numbers that give remainder 5 upon division by 8: 5, 13, 21, 29, 37, etc. all of these give remainders of 1 upon division by 4, so that's convincing enough. sufficient.
(note: the gmat WILL NOT give problems on which a spurious pattern appears, only to be broken after the 40th or 50th number; if you see a pattern persist for 4–5 cases, you can take it on faith that the pattern persists indefinitely.)

for (2), you should make the same realization you made above: one of the numbers has to be odd and the other even. then just try a bunch of possibilities:
1^2 + 2^2 = 5
2^2 + 3^2 = 13
3^2 + 4^2 = 25, etc
1^2 + 4^2 = 17
2^2 + 5^2 = 29
3^2 + 6^2 = 45, etc
all these give a remainder of 1 upon division by 4. sufficient.

16.
1. p can be represented as p = 8n+5
so p can take values , 13, 21, 29,37, 45.....just plugging in various values of integer n.

Now the next task is to represent all these odd numbers as sum of 2 perfect squares.

13 = 4+9 = 2^2+3^2 this implies x=2, y=3 . As question already told us that y is odd.
21 = cant represnt as sum of two +ve integers
29 = 4+ 25 = 2^2+ 5^2 ....implies x=2, y=5.
37 = 1+36 ....implies y=1 , x=6
45 = 9+36....implies y=3, x=6

So it tells us that x = 2, 6...not divisble by 4.

Hence SUFFICIENT

2. Condition B tells us
x–y= 3 and y is odd

so y=1 , x=4 Div by 4
y= 3 , x= 6 Not Div by 4
y= 5, x= 8 Div by 4
y= 7, x=10, Not DIv by 4
So INSUFFICIENT

17.
TAKEAWAY:

in REMAINDER PROBLEMS:
if you don't INSTANTLY see the algebraic solution, then IMMEDIATELY start LOOKING FOR A PATTERN.

there's also a fact that you should know concerning this problem statement:

fact:
REMAINDERS UPON DIVISION BY 10 are simply UNITS DIGITS.

for instance, when 352 is divided by 10, the remainder is 2.

since remainders are fundamentally based on stuff repeating over and over and over again, it shouldn't be a surprise that patterns emerge early and often among remainders.

this solution isn't necessarily "easier" – that judgment depends upon how comfortable you are with the algebra and theory – but it can be quite efficient.

using (1)
9*3^4n + m becomes 9*3^8 + m
considering only units digit , 9*1 + m
INSUFFICENT

instead, you can just realize that, since *m* can be anything at all, you can have any units digit you want.

using (2)
9*3^4n + 1 , as shown above, for all values of n, units digit 3^4n remains the same. (UD of 3^4=1,UD of 3^8 =1)

Now , considering only units digit
9*1 + 1 = 10 ,Hence B SUFFICIENT

yeah.

technically, you should also throw away the "1" in your sum of 10, reducing to a final units digit of 0.

The answer is 'B', but I don't get it!!! if m is one, you still don't know what 3^(4n+2) is... right? all we know is it's a power of 3... so it's units digit could be any number between 0–9.... thus, we still don't know what the remainder would be if divided by 10.... please help!!!!

Rephrase the expression: –
3^(4n+2) +m = (9)*3^(4n) + m

statement (1) n =2 so the expression = (9)*3^8 + m but we do not know what m is – so cannot predict the value of the expression. INSUFFICIENT

statemtn (2) m = 1 which makes the expression:

(9)*3^(4n) + 1 Since we know n is +ve integer, now it gets tricky: –

for n = 1,2,3,4 the exponential component of the expression will be

3^4, 3^8, 3^12 or
9^2, 9^4, 9^6 or

81, 81^2, 81^3 ans so on... the unit digit of all these values will be 1, now this value will be multiplied by 9 and '1' will be added to the result. It will make the unit digit of the result – 0. It means the result will be prefectly divided by 10. So the remainder will be 0

SUFFICIENT, So the answer is (B)

18.

(1)
if n = 3, then (n – 1)(n + 1) = 8, so the remainder is 8
if n = 5, then (n – 1)(n + 1) = 24, so the remainder is 0
insufficient

(2)
if n = 2, then (n – 1)(n + 1) = 3, so the remainder is 3
if n = 5, then (n – 1)(n + 1) = 24, so the remainder is 0
insufficient

(together)
the best approach, unless you're really good at number properties, is to try the first few numbers that satisfy both statements, and watch what happens.
if n = 1, then (n – 1)(n + 1) = 0, so the remainder is 0
if n = 5, then (n – 1)(n + 1) = 24, so the remainder is 0
if n = 7, then (n – 1)(n + 1) = 48, so the remainder is 0
if n = 11, then (n – 1)(n + 1) = 120, so the remainder is 0
...you can see where this is headed.

here's the theory:
– if n is not divisible by 2, then n is odd, so both (n – 1) and (n + 1) are even. moreover, since every other even number is a multiple of 4, one of those two factors is a multiple of 4. so the product (n – 1)(n + 1) contains one multiple of 2 and one multiple of 4, so it contains at least 2 x 2 x 2 = three 2's in its prime factorization.
– if n is not divisible by 3, then exactly one of (n – 1) and (n + 1) is divisible by 3, because every third integer is divisible by 3. therefore, the product (n – 1)(n + 1) contains a 3 in its prime factorization.
– thus, the overall prime factorization of (n – 1)(n + 1) contains three 2's and a 3.
– therefore, it is a multiple of 24.
– sufficient

answer = c

takeaway:
once you're established "insufficient", do not bother testing additional cases!
the fact that n = 2 and n = 5 are both of the form (3k + 2) is random coincidence.

two:
if you look at the treatment of the 2 statements together, i have included both (3k + 1) and (3k + 2)–type cases in that treatment. unlike statement (2) alone, the combination of the 2 statements turns out to be sufficient, so this time i *must* consider all of the possibilities.
therefore, i do.

if n is not divisible by 3, then exactly one of (n – 1) and (n + 1) is divisible by 3

if n – 1 is divisible by 3, then n has the form 3k + 1.
if n + 1 is divisible by 3, then n has the form 3k + 2.
both have been considered.

19.

We can rephrase the statement as such:

Is: n(n^2 – 1) divisible by 4?

Is N(N–1)(N+1) divisible by 4?

Is the product of three consecutive integers divisible by 4?

Final rephrasing:

Is N an odd integer or is N a multiple of 4?

Evaluate the statements:

1) n = 2k + 1, where K is an integer.

2K + 1 will give us an odd integer for N. (YES)

The problem I had was with plugging in 0 for K.
2(0) + 1 = 1 0x1x2 = 0 (OA: A **0 is divisible by every positive integer.**)

note the following:
the *only* way you will encounter this sort of query is if you *plug in your own numbers*. in other words, the official problems WILL NOT require you to decide the issue of whether 0 is divisible by *n* (for whatever *n*); they restrict the scope of divisibility problems strictly to positive divisors and positive dividends.

however, you should still know this fact, because, as you have seen here, you will often encounter "extra" questions like this as artifacts of plugging in your own numbers. therefore, even though the gmat won't test the concept directly, you may still have to rely on it to solve the problem because of your number plugging.

—

as long as we're at it, if you encounter "negative multiples" in your number plugging adventures, then yes, those are divisible too. for instance, –4 is divisible by 4, as are –8, –12, and the whole lot.

20.
<u>Method 1: Visual/Number Line approach.</u>
(1) r is 3 times farther away from 0 than m is. But we have no "distances" given, nor any info about sign (i.e. is m left or right of 0?)

(2) On a number line, put a dot at 12. Put two dots on either side of it for m and r. What can vary? The distance between m and r--they can be very close to 12, or both very far away. Also, we don't know whether m is the dot to the left or to the right of 12.

(1)&(2) together: We still don't know distances (from 12 or 0), or whether m is left or right of r.

We can either have (case A) $r = 18$ and $m = 6$ or (case B) $r = 36$ and $m = -12$.

<u>Method 2: Algebra approach</u>
(1) $r = +/-3m$
(2) $r-12 = 12-m$, or $r+m = 24$.
(1)&(2) together: $r+m= (+/-3m)+m = 24$. Either $4m = 24$ (i.e. $m=6$) or $-2m = 24$ (i.e. $m = -12$).

since the natural instinct is to try only positive values for m and r, this is a very tricky problem.

Statement (1) tells us that $r = 3m$ or $r = –3m$ (as either case would result in an r with an absolute value that is three times that of m). Insufficient. Eliminate AD from AD/BCE Grid.

Statement (2) tells us that $(r+m)/2 = 12$. Insufficient. Eliminate B from remaining BCE Grid.

By substituting each equation from Statement (1) into the equation from Statement (2), the statements together tell us that $3m + m = 24$, so $m = 6$ and $r = 18$, or that $–3m + m = 24$, so $m = –12$ and $r = 36$. As there are still two possible values for r, the correct answer is E.

OR

if you'd rather conceptualize it (which is always a good idea for number–line problems like this one), you can think of it this way:
r is 3 times as far away from 0 as is m, but we don't know in which direction.
that's the big thing.

since 12 is halfway between m and r, imagine m and r both starting out at 12, and 'sliding' equally in opposite directions, with r moving to the right and m moving to the left. (you can't slide r to the left and m to the right, because, if you do so, then r will be closer to 0 than is m.)
when the numbers have 'slid' a certain distance – specifically, 6 units each, so that $m = 6$ and $r = 18$ – they'll arrive at a point where the distance between m and 0 is 1/3 of the distance between r and 0. that's the first point that satisfies both criteria.
now keep sliding the points away from 12.
eventually, m will pass through 0 itself, and will come out on the negative side. if you keep sliding, you'll reachanother point at which the distance from 0 to m is 1/3 of the distance from 0 to r, only this time m is negative. (specifically, this will happen when $m = –12$ and $r = 36$.)

21.

the OA is C

Consider Data 2 independently

We have two possibilities:
1) Keep S on the right side of zero and satisfy the condition
2)Keep S on the left side and again satisfy the condition

In both ways the data is sufficient, but our quest is whether zero is halfway ...this is where u seem to have miss out...

If u dont consider the Data 1 then u may or may not get zero halfway...

Thats y the answer is C

<——————————R——————S———T———————>

OR

Choice (B) does not eliminate the possibility that R & S are zero. Combining the two statements eliminates zero as an answer and gives us a definite "yes" as an answer.

watch those assumptions.

the distance between t and (–s) must be a positive number, but the problem is that we don't know which way to subtract to get that positive number. if t > –s, then the distance is t – (–s), as you've written here. however, if –s > t, then the distance is actually (–s – t) instead.
if s is to the left of zero, then –s will be to the right of zero – which could well place –s to the right of t. if that happens, then the distance will become (–s – t), rendering your calculation inaccurate. try drawing out this possibility – put zero WAY to the right of both s and t on the number line, then find –s, and watch what happens).

if s lies to the right of zero, then –s must lie even further to the left than does s itself. since s is already to the left of t, it then follows that –s is also to the left of t. therefore, in that case, you can definitively write the distance as t – (–s), and your calculation is valid. therefore, (c).

—

ironically, the presence of statement (1) should make it *easier* to see that statement (2) is insufficient. specifically, statement (1) calls your attention to the fact that s *could* lie to the *left* of zero, in which case you could get the alternative outcome referenced above. that's something you might not think about if statement (1) weren't there.

plug in numbers to the number line here:

Statement 1)
if the line reads: r=–1, zero, s=1, t=3, then zero is halfway between r and s.
if the line reads: zero, r=1, s=2, t=3, then zero is not between r and s.
Insufficient.

Statement 2)
by definition zero is halfway between s and –s.

a) if the line reads: –s=r=–2, zero, s=2, and t=4, then (t to r)=(t to –s)=6.
zero is halfway in between r and s.

b) if the line reads: r=–4, s=–2, t=–1, zero, and –s=2, then (t to r) = (t to –s) = 3.
zero is between t and –s.
Insufficient.

Together)
Forces the case 2a). Sufficient.

OR

the particular trap you may have fallen into in your interpretation of (2) is that of assuming "–s" is to the LEFT of "t". there is no good reason whatsoever to make this assumption, and, what's more, at least one good reason (viz., "the gmat loves to test exactly these sorts of assumptions) not to make it.

of course, you don't need reasons to be very careful about your assumptions; that should be your default state.

if "–s" is to the right of "t", then you have
<—r————s——t———————(–s)—>
in which case 0 is in no–man's–land between "t" and "–s".
in this case, note that "s" is negative. also note that **(–s) is positive** in this case, a situation that is difficult to digest for most students.

taking statements (1) and (2) together eliminates the above possibility, leaving only the case that you have outlined.

—

incidentally, the fault in the algebraic approach lies in writing the distance between t and (–s) as t – (–s). this writing is correct only if t is greater than (–s), an assumption that, as we've seen, is unjustified.
the correct way to write the distance is $|t - (-s)| = |t + s|$, an expression that is thoroughly unhelpful in solving this problem.

22.
The answer is A.

S and t are different numbers on the line segment, Is s+t=0?
We need to know where s and t are in the line segment

Using BDACE Grid ,

2 says 0 is between s and t
In a line segment s and t are two points and 0 is between them. Let says s at –7 in the coordinate, t could be in 3 and 0 is between them. It does not give a statement that s+T=0. Insuff

1 says distance between s and o is = d(betwen t and o)
Clearly, 0 is between S and t because distance from s to 0 is equal to distance from t to 0.
This gives a way to solve for s+t=0. Hence A is sufficient

Statement 2 is insufficient because 0 is between s and t. But that means s can equal –5 and t can equal +3. In such a case, 0 is still between s and t but that does not make them equidistant from 0. Or, s and t can be –4 and +4 respectively in which case they are equidistant from 0. Therefore, this statement doesn't necessarily answer the question because it can have different results.

The question states that s and t are different numbers, so they cannot both be –5. Therefore, they must be opposites of each other.

just as in normal parlance, "between" only means "between", and carries no connotations of equidistance from the two points.
for instance, it's quite true that 1 is between 0 and 100, but obviously false that 1 is the midpoint between 0 and 100.

same thing with statement two. if 0 is between s and t, then all this means is that one of s and t is positive and the other is negative. that is all; there's nothing barring possibilities such as s = –1,000,000 and t = 1.

23.

well, first, think about the qualitative aspects of the sequence: if the sequence consisted entirely of 7's, then there would be fifty terms in the sequence. these answer choices are reasonably close to fifty, so it stands to reason that by far the majority of the terms will be 7's. therefore, try as few 77's as possible.

try only one 77:
remaining terms = 350 - 77 = 273
this would be 273 / 7 = 39 sevens
so ... you'd have one '77' and thirty-nine '7's

this works!

answer = c

OR

Since the units digit of 350 is zero, you know that the number of terms in the equation must be such that:

n*7 = number with units digit of zero

The only time this is true is if n is 10 or a multiple thereof, and 40 is the only answer that satisfies that.

24.
after the first two terms i.e $(2^2)+(2^3)+(2^4)+(2^5)+(2^6)+(2^7)+(2^8)$ is a GP series with first term as 2^2 and ratio as 2.
Using the formula for sum of GP series, for this part, the original equation becomes
$2^2 + 2^2(1–2^7)/(1–2)$
$= 2^2 + 2^2(127)$
$= 2^2(1 + 127)$
$= 2^2 * 2^7$
$=2^9$

OR

there are several ways.

(1) **PATTERN RECOGNITION**
it should be clear that there's nothing special about 2^8 as an ending point; in other words, they just cut the sequence off at a random point. therefore, if we **investigate smaller "versions" of the sequence, we should be able to detect a pattern**.
let's look:
first term = 2
sum of first 2 terms = 4
sum of first 3 terms = 8
sum of first 4 terms = 16
ok, it's clear what's going on: each new term doubles the sum. **if you see a pattern this clear, it doesn't matter whether you understand WHY the pattern exists; just continue it.**
so, i want the sum of nine terms, so i'll just double the sum five more times:
32, 64, 128, 256, 512.
this is choice (a).
this is a general rule, by the way: IF SOMETHING CONTAINS MORE THAN 4–5 IDENTICAL STEPS, YOU SHOULD BE ABLE TO EXTRACT A PATTERN FROM LOOKING AT SIMILAR EXAMPLES WITH FEWER STEPS.

(2) **ALGEBRA WITH EXPONENTS** ("textbook method")

the first two terms are 2 + 2. this is 2(2), or 2^2.
now, using this combined term as the "first term", the first two terms are 2^2 + 2^2. this is 2(2^2), or (2^1)(2^2), or 2^3.
now, using this combined term as the "first term", the first two terms are 2^3 + 2^3. this is 2(2^3), or (2^1)(2^3), or 2^4.
you can see that this will keep happening, so it will continue all the way up to 2^8 + 2^8, which is 2(2^8) = (2^1)(2^8) = 2^9.

(3) ESTIMATE
these **answer choices are ridiculously far apart**, so you should be able to estimate the answer.
memorize some select powers of 2. notably, 2^10 = 1024, which is "about 1000". 2^9 = 512, which is "about 500". and of course you should know all the smaller ones (2^6 and below) by heart.
thus we have 2^8 is about 250, and the other terms are 128, 64, 32, 16, 8, 4, 2, 2.
looking at these numbers, i'd make a ROUGH ESTIMATE WITHIN A FEW SECONDS:
250 is 250.
128 is ~130.
64 and 32 together are ~100.
the others look like thirty or so together.
so, 250 + 130 + 100 + 30 = 510.
the only answer choice within shouting range is (a); the others are absurdly huge.

—

even if you have no idea how to do anything else, **you should still be able to do out the arithmetic within the two–minute time limit.**
it won't be fun, but you should be able to do it. if you can't, then the reason is probably "you stared at the problem for too long, and didn't get started when you should have".

yes ,the shortest method on this planet to solve the above question.

This formula may be of use:2^1+2^2+........+2^n =[2^(n+1)] – 2, where n equals to number of terms.

question is: 2+2+(2^2)+(2^3)+(2^4)+(2^5)+(2^6)+(2^7)+(2^8)

this can be written as : 2+(2^1)+(2^2)+(2^3)+(2^4)+(2^5)+(2^6)+(2^7)+(2^8)

Therefore, 2+[2^(8+1)] – 2 = 2^9 (answer)

25.
OA is C

statement (1) means that the smallest and largest elements of the list have the same sign, i.e., are both positive or both negative.
but, since those are *the smallest and largest elements of the list*, that means that all the elements *between* have to have that same sign, too.
or:
you can't have 0 between two positive numbers, or between two negative numbers.
either *everything* in the list is positive, or *everything* in the list is negative.

From statement (1), we know the product of the highest and the lowest integer is + ve, it means either both of them are +ve or –ve

For ex: –2,–1,1,2 in this list the product of –2&2 is –4. It proves both the highest and the lowest terms have to be of same sign.

(+ve or –ve) the other factor that needs to be considered is the no. of terms in the list,

If the no. of terms is odd and all the integers are –ve, the product of all the integers will be –ve this information is given by statement (2)

no. of terms in the list are even, hence the product of all the integers in the list will always be +ve.

So if you combine (1) & (2), they are sufficient.

You have to multiply the smallest and the largest to satisfy case I. For exmple {+,–,–,+} does not satisfy case I. You have taken both negative numbers in the middle. But the smallest number will be one of the negative numbers and the largest one of the positive ones, giving a negative product of as opposed to positive. Same holds for the third example you have used.

if you have numbers arranged from least to greatest, then any '–' numbers must show up to the left of *all* '+' numbers. otherwise, you've created an impossible situation in which a negative number is somehow bigger than a positive number.

26.
(1) imagine 0, 0, 0, 0 OR 0, 0, 0, 2 NS
(2) in order for the sum of "any 2 numbers" to = 0, all the numbers must equal 0
SUFFICIENT

statement 2 gives: the sum of ANY TWO...
So, since there are "more than 2" numbers in the set, the set contains (in your example): (–2, 2, x). because the set contains at least that x, the sum of "any 2" numbers, for instance 2 + x, does

not have to equal zero.
so, INSUFFICIENT

you need to have more than 2 numbers in the set. the problem is that ANY two numbers have to sum to zero – which means that if you pair the mystery third number with EITHER of the existing two numbers, you must get a sum of zero.

if your first two numbers are 2 and –2, that's impossible: there's no number that will add to 2 to give zero, and will ALSO add to –2 to give zero.

in general, if your first two numbers are –x and x, then your third number must be x (so that it adds to –x to give zero), but it must ALSO be –x (so that it adds to x to give zero). the only way that x can equal –x is if x is zero – which means that all three numbers are zero.

therefore, everything must be zero.

27.

The answer is A
With statement 1:
this function can only be addition or multiplication
with either of these two operations the left side does indeed equal the right...sufficient
With statement 2
this function can be either multiplication or division
with multiplication the left and right side equal one another
with division it doesn't...
hence 2 is insuffcient.

note the general takeaway here:
if you have a problem like this, in which a mystery symbol stands for one or more of a collection of operations, then your #1 goal is to figure out ANY AND ALL operations for which that symbol can stand.

28.

Let's consider (1) N+1 >0. This clearly tells you nothing about p, so is insuffienct by itself, ruling out A&D.

Let's look at (2). np >0. This tells us that n,p and either both positive or both negative. Therefore it is insufficient to answer whether p >0, so we can eliminate B.

Now let's consider (1) and (2) together. (1) combined with the fact that N and P are integers tells us that N>= 0. (2) tells us that N and P are either both positive or both negative and that neither are equal to 0. Combined with (1) we therefore know what N is positive, and from (2) P must be positive too. So (1) and (2) together are sufficient and the answer is C.

29.

(2)
Takeaway #1: when you plug numbers on a DS problem, YOUR GOAL IS TO PROVE THAT THE STATEMENT IS INSUFFICIENT.
Therefore, as soon as you get a 'yes' answer, you should be TRYING to get a 'no' answer to go along with it; and, as soon as you get a 'no' answer, you should be TRYING to get a 'yes' answer to go along with it.
Statement (2)
you need to pick numbers such that $x + y > z$, per this statement.
First, pick a completely random set of numbers that does this: how about $x = 1$, $y = 1$, $z = 0$. These numbers give a YES answer to the prompt question, since 1^4 + 1^4 is indeed greater than 0^4. Now remember: **your goal is to prove that the statement is INSUFFICIENT.** This means that we have to try for a 'no' answer. This means that we have to make z^4 as big as possible, while still obeying the criterion $x + y > z$. Fortunately, this is somewhat simple to do: just make z a big negative number. Try $x = 1$, $y = 1$, $z = -100$. In this case, $x + y > z$ (satisfying statement two), but x^4 + y^4 is clearly less than z^4, so, NO to the prompt question. Insufficient.
Statement (2)
you need to pick numbers such that x^2 + y^2 > z^2, per this statement. First, pick a completely random set of numbers that does this: how about $x = 1$, $y = 1$, $z = 0$ (the same set of numbers we picked last time). These numbers give a YES answer to the prompt question, since 1^4 + 1^4 is indeed greater than 0^4. Now remember: **your goal is to prove that the statement is INSUFFICIENT.** This means that we have to try for a 'no' answer. This means that we have to make z^4 as big as possible, while still obeying the criterion x^2 + y^2 > z^2. Unfortunately, this isn't as easy to do as it was last time; we can't just make z a huge negative number, because z^2 would then still be a giant positive number (thwarting our efforts at obeying the criterion). So, we have to finesse this one a bit, but the deal is still to make z as big as possible while still obeying the criterion. Let's let x and y randomly be 3 and 3. Then x^2 + y^2 = 18; we need z^2 to be less than this, but still as big as possible. So let's let $z = 4$ (so that z^2 = 16, which is pretty close). With these numbers, x^4 + y^4 = 162, which is much less than z^4 = 256. Therefore, NO to the prompt question, so, insufficient. Answer = e.

Takeaway #2: if a statement is sufficient, then you WILL be able to PROVE that it is, algebraically or with some other form of theory.
In other words, you'll never get a statement that's sufficient, but for which you can only figure that out by number plugging.
It's obvious that you can get a YES answer to the question; all you have to do is take ridiculously big numbers for x and y, and a small number for z. for instance, $x = y = 100$, $z = 0$, satisfy both statements, and clearly give a YES answer. So, you're trying for a NO answer. Try to make Z as big as possible while still satisfying the criteria (i.e., less than x^2 + y^2). Let's let $x = y = 3$ then to satisfy both statements, we need z^2 less than 18, and z less than 6. We'll take $z = 4$,

which is pushing the limit of the first one. In this case, then, x^4 + y^4 = 81 + 81 = 162, but z^4 = 256, giving a NO answer. Insufficient   Answer = e

30.
Ans. A
(1).. add 1 to both sides… you get r > w.
(2) gives contradictory answers… NS

31.
We're told x and y are positive but not whether they are greater than 1, so I have to consider fractional possibilities. How do I know what to try?

When I take a square root:
Anything greater than 1 will get smaller (but remain larger than 1)
1 will stay the same
Anything between 0 and 1 will get bigger (but remain a fraction between 0 and 1)

When I take a reciprocal in each of the above cases:
1/something larger than 1 = something smaller than 1 (but still positive)
1/1 = 1
1/something smaller than 1 = something larger than 1

If I want to try numbers now, then I know I need to try a number from each set. Or I can continue with logic and the algebraic representations. Do whichever you are most comfortable with.

For trying numbers, first try something greater than 1:
x=2, y=2 (I'm trying the same numbers b/c I'm trying to see if I can prove things false and funny things happen when you use the same number for different variables). 1/(4)^.5 = 1/2.
Roman Numeral 1 (RN1): (4)^.5 / 2(2) = 2/4 = 1/2. Same, not greater, so elim RN1.
RN2: (2^.5 + 2^.5) / (4) = 2(2^.5) / 4. Well, 2^.5 is about 1.7. 2*1.7 = 3.4 / 4 = more than 1/2. So RN2 is okay, at least with this instance.
RN3: (2^.5 – 2^.5) / 4 = 0/4 = 0. Elim RN3.

At this point, I don't know whether I have to try more numbers b/c the answer choices haven't been listed. If I have both "none" and "II only" as options, then I have to try more numbers. If "none" is not an option, then I'm done.

You will notice that equation 2 will always be more .
THE FASTEST WAY IS TO EXPRESS EACH EQUATION AS A FUNCTION OF 1/(X+Y)^0.5 . This aproach takes less than a minute.

there's little sense in dealing with #3 algebraically: because of the subtraction, it can clearly equal 0 (if x and y are the same number). since 1/√(x + y) is a positive number, the possibility of 0 rules out roman numeral III. (in fact, that expression can even be negative, as nothing prohibits x from being smaller than y.)

if you want to **compare two fractions**, you can use the technique of **cross products** to perform the comparison.
to use this technique, you take the two 'cross products' (one of the numerators, times the denominator of the other fraction), and associate each of the cross products with whichever fraction donated the numerator.
for instance, if you're comparing 2/3 vs. 11/17, then the cross products are 2 x 17 = 34 (associated with 2/3) and 3 x 11 = 33 (associated with 11/17). because 34 is greater than 33, it follows that 2/3 is greater than 11/17.

notice that this technique only applies to **positive** fractions... but that's all you really need: if the fractions have opposite signs, then the comparison is trivial (the positive one is bigger!), and if the fractions are both negative, then the comparison is the opposite of whatever it would be if they were positive.

find cross products in #(i):
$\sqrt{(x + y)}/2x$ vs. $1/\sqrt{(x + y)}$
cross products are $(x + y)$ vs. $2x$
subtract one x from both sides —> this comparison is the same as y vs. x
we don't know which is bigger.

find cross products in #(ii):
$(\sqrt{x} + \sqrt{y})/(x + y)$ vs. $1/\sqrt{(x + y)}$
cross products are $(\sqrt{x} + \sqrt{y})\sqrt{(x + y)}$ vs. $(x + y)$
divide both sides by $\sqrt{(x + y)}$ to give $(\sqrt{x} + \sqrt{y})$ vs. $\sqrt{(x + y)}$ —– remember that (quantity) divided by √(quantity) is √(quantity) — that's the definition of what a square root is.
since both of these quantities are positive, we can square them and compare the squares:
$(\sqrt{x} + \sqrt{y})^2$ vs. $(\sqrt{(x + y)})^2$
$x + 2\sqrt{xy} + y$ vs. $x + y$
left hand side is bigger
so the original fraction is bigger than $1/\sqrt{(x + y)}$

ans = ii only

32.
OA: B

1. $|x - 3| \geq y$
Taking numbers:
x: –2, 1 both can satisfy the above equation. Insufficient.

2. $|x - 3| \leq - y$

Since $|x-3|$ is an absolute value, the smallest it can go is 0.
And since y is given to be >0, thus – y will give a negative value which will cause the equation to fall apart unless it is 0.

so $|x-3| = 0$.
$x = 3$.

33.
OA: C

we can rephrase this to $x = z - y$.
there are thus 3 possibilities for the absolute value $|x|$:
(a) if $z - y$ is positive, then $|x| = z - y$, and will NOT equal $y - z$ (which is a negative quantity).
(b) if $z - y$ is negative, then $|x| = y - z$ (the opposite of $z - y$).
(c) if $z - y = 0$, then $|x|$ equals both $y - z$ and $z - y$, since each is equal to 0.
TAKEAWAY: when you consider absolute value equations, you'll often do well by considering the different CASES that result from different combinations of signs.
notice that (a) and (b), or (a) and (c), taken together prove that statement 1 is insufficient.

statement 2:
we don't know anything about y or z, so this statement is insufficient.**
if you must, find cases: say $y = 2$ and $z = 1$. if $x = -1$, then the answer is YES; if x is any negative number other than –1, then the answer is NO.

together:
if $x < 0$, then this is case (b) listed above under statement 1.
therefore, the answer to the prompt question is YES.
sufficient.

—

WE could craft a statement that doesn't mention all three of x, y, z and yet IS STILL SUFFICIENT.
here's one way we could do that:
(2) $y < z$
in this case, $y - z$ is negative and therefore CAN'T equal $|x|$ — no matter what x is — since $|x|$ must be nonnegative.
so, this statement is a definitive NO, and is thus sufficient even though it doesn't mention x at all.

34.
Ans. C… standard property.

35.
If $x = 0.1$, then $x^2 < 2x < 1/x$ (so 1 is possible)

If $x = 0.9$, then $x^2 < 1/x < 2x$ (so 2 is possible)

(1) $x^2<2x<1/x$

This means that $x^2<2x$ so divide by x to get $x<2$. The second one tells you that $2x<1/x$ which simplifies to $x < 1/\text{sqrt}(2)$. These can obviously both be satisfied at the same time, so (1) works.

(2) $x^2<1/x<2x$

This means that $x^2<1/x$ which gives $x^3<1$, or $x<1$. The second half gives you $1/x<2x$ or $1<2(x^2)$ or $x>1/\text{sqrt}(2)$. So any number that satisfies $1/\text{sqrt}(2)<x<1$ will work.

(3) $2x<x^2<1/x$. The first part gives $2x<x^2$ or $x>2$. The second half gives $x^2<1/x$ or $x^3<1$ or $x<1$. Since the regions $x>2$ and $x<1$ do not overlap, (3) can not be satisfied.

The Answer choice is (4), 1 and 2 only

> if $x = 1/2$, then:
> $x^2 = 1/4$
> $1/x = 2$
> $2x = 1$
> so the order would be $x^2 < 2x < 1/x$. So (a) is possible. Eliminate i and iii. (Looks like you got this far)
>
> if $x = 3/4$, then:
> $x^2 = 9/16$
> $1/x = 4/3$
> $2x = 3/2$
> so the order would be $x^2 < 1/x < 2x$. So (b) is possible. Eliminate ii. (Looks like this is where you had trouble.)

36.
Notice that we're dealing with a fractional inequality, which, worse yet, CAN'T be multiplied by the common denominator (since we don't know the sign of that denominator).
Therefore, **pick numbers**.
Just be careful to pick APPROPRIATE numbers. i.e., this problem contains **sums and differences**, as well as **sign considerations**, so i would pick:
* POSITIVES AND NEGATIVES (as allowed by the statements)
* DIFFERENT RELATIVE SIZES (i.e., "bigger" and "smaller" numbers) — this is important because of addition and subtraction.
so, for statement (1), i would pick:

1, 2
2, 1
1, –2
2, –1
1, 0
for statement (2), i would pick:
1, –2
2, –1
–1, –2
–2, –1
0, –1
for "together" i would look at the two common elements, which are (1, –2) and (2, –1).
note that this is a lot of plug–ins, but you don't wind up trying them all – you STOP as soon as you get "insufficient".

GMAT's answer is (E)

this is a difficult problem, because it resists simple algebra. **you CANNOT multiply through by the denominator (x + y), because the sign of that denominator is unknown.**

therefore, you have to leave the problem as written (ugly as it may be).

since there's no simple algebraic solution, a fallback is to **recognize the types of numbers that are important in the problem, and try numbers across those categories.**

there are two things that matter in this problem (as may be deduced from an inspection of the problem + experience with these sorts of things):
1. positive vs. negative
2. the relative magnitudes of x and y

let's try numbers across both of these categories.

statement (1)
x must be positive, but y could be positive or negative, and smaller or bigger (or the same) in magnitude.
if x = 1 and y = 2 —> answer = NO
if x = 2 and y = 1 —> answer = NO
if x = 1 and y = –2 —> answer = NO
if x = 2 and y = –1 —> answer = YES
insufficient
[at this point you could notice that the last two examples also satisfy statement 2, and therefore satisfy statements 1 and 2 together. this fact proves that the answer is E right now, and you're done. if you don't notice this (most students won't), then go on.]

statement (2)
y must be negative, but x could be positive or negative, and smaller or bigger (or the same) in

magnitude.
if $x = -1$ and $y = -2$ —> answer = NO
if $x = -2$ and $y = -1$ —> answer = NO
if $x = 1$ and $y = -2$ —> answer = NO
if $x = 2$ and $y = -1$ —> answer = YES
insufficient

together
x must be positive and y must be negative, but the relative magnitudes can go any way (bigger/smaller/same)
if $x = 1$ and $y = -2$ —> answer = NO
if $x = 2$ and $y = -1$ —> answer = YES
insufficient

ans (e)

37.
Let us take statement I – In this, it is given that $1/(k-1) > 0$. This implies that k must be postive and k must be greater than 1. Hence, $1/k$ is definitely greater than zero. For example, k's value is 2, then $1/(2-1) = 1$ which is > 0. This implies that $1/2 = 0.5$ which si still greater than '0'. Hence, this is sufficient.

Let us take II – It says $1/(k+1) > 0$ which means that this will satisfy for both positive and negative values of k which are > than -1. for example, if k is 2, $1/(k+1)$ is >0 and $1/k$ wll be >0. But if k's value is -0.5, it will satisfy the second equation but $1/k$ will be -2 which is <0 and hence, INSUFFICIENT.

Hence, A alone is sufficient to answer this question.

38.
The sign of a variable has nothing to do with addition and subtraction.
It is only MULTIPLICATION AND DIVISION that are affected by the sign of the quantity.
There's a much worse problem here: you CAN'T SUBTRACT INEQUALITIES that face the same way. You can add them, but you can't subtract them.
The answer is B

if you don't have an approach, then you should immediately start plugging in. you should do ANYTHING to ensure that you're not just sitting there staring at a problem.

—

Statement (2):
All you need is $y < w$, which is EXACTLY equivalent to $w - y > 0$. There are two ways you

could figure this out:
So #2 is sufficient. they're just including x in there to try to get you to waste your time.

You cannot subtract two inequalities that face the same way.

Think about this:
$x < 10$
$y < 10$
if you try to subtract these, then you'll get $x - y$ (?) 0.
but that clearly doesn't work, since you could create possibilities for "<" (e.g. $x = 7$, $y = 8$); "=" (e.g., $x = y = 8$); or ">" (e.g., $x = 8$, $y = 7$).

Incidentally, if two inequalities **face in OPPOSITE ways**, then you can subtract them. But if that's the case, it's easier to just **multiply one of them by –1 and then add them**.

The question is asking is $w-y>0$

And After reading the the St.1 , things you know are
$w+x < 0$ from Question Stem &
$x+y < 0$ from St.1
Nothing more is given, don’t use the inequality used in the Question.
So the above statements can be true
$w=1$, $y=2$ and $x= -10$
$w+x = -9<0$
$w+y = -8 <0$

or
$w=2$, $y=1$, $x=-10$
$w+x = -8<0$
$w+y = -9<0$

And as you can see we cant say whether $w>y$ or $w<y$ and hence insufficient.

39.
OA: A
The quick way to approach will be pick a number $x<0$
Lets pick –5
so we know $x=-5$
sqrt $(-x|x|)$ = sqrt $(-(-5)|-5|)$
= sqrt $(5*5)$ = sqrt $(25) = 5 = -(-5) = -x$
So Answer A.

40.
If statement 2 contains statement 1, then ELIMINATE (b) and (c).
If statement 1 contains statement 2, then ELIMINATE (a) and (c).

Ans– D (BOTH SUFFICIENT)

SQRT ((x–5)^2 = |x–5|
squaring a quantity, and then square–rooting, is equivalent to taking the absolute value.

we can make this more clear:
|x – 5| can be either (x – 5), the actual quantity within the absolute–value bars, or (5 – x), the opposite of that quantity.
if it's to be the original quantity (x – 5), then that quantity must be at least 0: $x \geq 5$.
if it's to be the opposite (5 – x), then that opposite quantity must be at least 0. for that to happen, $x \leq 5$.
(notice that, if x is actually 5, then |x – 5| equals *both* (x – 5) and (5 – x), since both of them are zero.)

therefore, we can rephrase the question:
is $x \leq 5$?
when you see this statement, it may bewilder you at first, but you should look at it and think: "ok, just absolute–value bars and negative signs. no other numbers; no other operations; this could only possibly have to do with the sign of x."
then just test it to see whether it works for PNZ (positive, negative, zero).
turns out that it only works for negative numbers.
therefore, rephrase:

1. $-x|x| > 0$

the above is possible only for $x < 0$, therefore $(x-5) < 0$

$|x-5| = 5-x$ this becomes – – – > $-(x-5) = 5-x$ and hence sufficient

2. Clearly states that $(x-5) < 0$ so this is sufficient and the answer is D

(1) $x < 0$

this is sufficient, since x is definitely less than 5 if it's negative.

41.
the absolute value will do one of two things to a quantity:
(a) LEAVE THE QUANTITY ALONE, if the quantity is POSITIVE;
(b) REVERSE THE SIGN of the quantity, if the quantity is NEGATIVE.
if the quantity is exactly 0, then both of these result in the same number, so it doesn't matter which of them you call it.

therefore:

the expression $|x - 3|$ will equal one of two expressions:
LEFT ALONE as $(x - 3)$, if $x - 3$ is POSITIVE — i.e., if x is greater than 3;
REVERSED to $(3 - x)$ (which is the same as $-x + 3$), if $x - 3$ is NEGATIVE — i.e., if x is less than 3;
EITHER of these (since both equal 0) if x is exactly 3.

therefore, we now have a rephrase of the question.
REPHRASE:
is $x \leq 3$?

so, the answer is (b).

you can also solve this problem, perhaps more easily, by **PLUGGING IN NUMBERS**.

statement (1):
since 3 is clearly a pivotal number in this problem, try numbers that are greater than 3 and numbers that are less than 3.

try $x = 0$:
$\sqrt{((x - 3)^2)} = 3$
$3 - x = 3$
answer to prompt question = YES

try $x = 5$:
$\sqrt{((x - 3)^2)} = 2$
$3 - x = -2$
answer to prompt question = NO

insufficient.

—

statement (2)
this statement is an obnoxious way of stating that x is a negative number. (you still have to figure that out — no way around it)

if you try plugging in a vast array of negative numbers – big, small, even, odd, etc. – you'll find that the equality in the prompt question holds for ALL of them.

sufficient.

42.

A

When multiplying or dividing an inequality by a negative, we have to switch the sign. We're not told whether the quantity (a–b) is pos or neg, so we have to check this both ways.
IF a–b is pos, then 1 < (a–b)(b–a) (and you can simplify further from there)
IF a–b is neg, then 1 > (a–b)(b–a) (and again you can simplify further from here)
But the point is that I have to follow both possibilities through and any particular statement is only sufficient if BOTH equations give me the SAME definitive answer, yes or no.
I'll stop there – try this again and come back if you have more questions.
Is 1/(a–b) < b – a?

Rephrase: 1< (a–b) (b–a)

1) a < b

If a, b are both +ve, Both –ve, or –ve & +ve

1 > (a–b) (b–a)
Statement (1) is sufficient.

2) 1 < |a–b|

This statement mentions |a–b| and nothing about (b–a)

(b–a) can be +ve or –ve.

Therefore, statement (2) is not sufficient.

43.
if you have
| QUANTITY 1 | = | QUANTITY 2 |
(with NOTHING ADDED to, or SUBTRACTED from, the abs. values)
then
just SOLVE TWO EQUATIONS:
*** QUANTITY 1 = QUANTITY 2**
*** QUANTITY 1 = –(QUANTITY 2)**
this is all you need. (there are also the possibilities with a negative sign in front of quantity 1, but those will just be equivalent the two already written here.)

statement (1)
for EQUATIONS involving absolute value, like this one, the key realization is that the absolute value of a quantity can signify either the quantity itself or the *opposite* of the quantity. therefore, if you try each of the sign combinations (pos/neg) of the absolute values in the problem, you'll be guaranteed to find all of the solutions.
(note: in what follows, "+" means leaving the expression within the absolute value bars alone; "–" means reversing the sign of that expression)
in this equation, there are ostensibly four sign combinations, +/+, +/–, –/+, –/–, but it's only

necessary to try two of them:
** first, either +/+ or –/–, in which *both* or *neither* of the absolute value expressions are flipped. may as well go with +/+ (i.e., leaving both of the absolute value expressions alone while removing the bars): x + 1 = 2(x – 1), or x = 3. plugging this back into the original equation shows that it works.
** second, either +/– or –/+, in which *one* of the absolute value expressions is flipped. let's go (at random) with flipping the first one: –x – 1 = 2(x – 1), or x = 1/3. plugging this into the original equation also shows that it works.

therefore, **statement 1 means that x = 3 or x = 1/3**.

statement (2)

two ways to interpret absolute value inequalities like this one:
** *memorize the template of the solution* (preferred for efficiency's sake): you should just know that |*expression*| > *a* means "either *expression* > *a* or *expression* < –*a*".
** *conceptualize absolute value as distance*: in this case, |*x* – 3| means the distance between *x* and 3. therefore, this statement means that the distance between x and 3 is greater than 0 (in either direction).
either of these interpretations means that *x* < 3 or *x* > 3, or, equivalently, ***x* is not equal to 3.**

statement 1 is insufficient, because 1/3 gives a "yes" and 3 gives a "no". statement 2 is also insufficient, because *every* number except 3 is possible. taken together, though, the two statements are sufficient because they yield a unique value, 1/3, for x.
notice that there's no reason even to figure out whether 1/3 gives "yes" or "no" at this point; it's *one value*, meaning that it is guaranteed to be sufficient no matter what the answer.

answer = c

44.
I is definitely true.

For II. z > y > x. Consider, x=1/4, y=1/3, z=1/2. This satisfies given primary inequality. Hence, II also holds true.

For III. Consider x=1, y=1/3,z=1/2. This satisfies given primary inequality. Hence, III also holds true.

Hence, E.

basically, the idea is that fractions (i.e., numbers between 0 and 1) "act funky" when they're raised to powers.
so do negatives.
therefore, when you pick numbers, you MUST consider these sorts of numbers!

you can take 3 cases,
1. When X,Y,Z are positive integers in that case:
I. X > Y> Z holds true

2. When X, Y, Z are fractions:
Z > Y > X
1/2 > 1/3 > 1/4
X>Y^2>Z^4
1/4 > 1/9 > 1/16

3. X > Z > Y (Negative, positive)
20 > –3 > –4
X>Y^2>Z^4
20 > 16 > 9

Therefore answer is E.

45.
Take –4,–3,–2–,–1 as the 4 consecutive integers. The answer is A

the numbers can be –4,–3,–2,–1 as well but do we know that those are the numbers?

–2,–1,0,1.. then 0 > –1, however, if the numbers were 1,2,3,4 then 3<8.

If p, q, r, and s are consecutive integers, with p < q < r < s and pq < rs;then r>0.5 and cannot be 0. In fact pr<qs making statement 1 sufficient.
Statement 2 give no information other than the universal truth 2>0.
The correct answer is indeed A.

46.
Statement 1 is sufficient.
Stmt 2 is y < 1 , since y is an integer , it can be 0 only if it is less than 1.
So Stmt 2 is sufficient too.
Both statements are sufficient enough to deduce if y = 0 .
Answer is D

47.
M – 3Z > 0
–M + 4Z > 0

if you add the two equations, M cancels off and you are left with only one Z which is practical :

M–M –3Z +4Z > 0

gives :
Z>0

NOTE : obviously you can do this only if the two inequality signs face in the same direction!

So we now Z is positive. Now consider "M – 3Z > 0". We can conclude M is positive

so M + Z ===> positive + positive so is positive

Thus C.

48.
OA is D.
believe it or not, it turns out that **JUST THE PROBLEM STATEMENT IS ALREADY SUFFICIENT** on this problem!

in other words, this problem is already "sufficient", **even without EITHER of the two statements!**

yes, you read that correctly.

proof:

* "zy < xy < 0" means that z and y have opposite signs, and x and y have opposite signs.
therefore, x and z have the same sign.
furthermore, z must be farther away from zero than x (because the magnitude of zy is greater than the magnitude of xy).

therefore, there are only 2 possibilities (shown on number line):

y————0————x—————z

or

z————x————0————y

now let's turn to the problem statement.

|x – z| is the distance between x and z.
|x| is the distance between 0 and x.
|z| is the distance between 0 and z.

using these interpretations, it's plain that |x – z| + |x| = |z| is ALREADY true for both of these statements.

neither of statements (1) and (2) is necessary.

technically, there's no answer choice that does this ("the problem statement is already sufficient"), although it's clear that you should pick (d) should this situation ever arise on the real exam.

There are different breeds of absolute value problems, so (as usual) there's no one neat, solid answer to a question like that. however:
* if a problem contains the symbols "> 0" or "< 0" at **any point**, you can rest assured that the crux of the problem involves the **signs** of quantities. (the problem in this thread is a perfect example.)
if you encounter such a problem, you should immediately devote all of your energy to rephrasing the question prompt and/or statements to equivalent formulations involving 'positive'/'negative'.
for instance, if you see
zy < xy < 0
you should think:
* z and x have the same sign
* y must have the opposite of whatever sign those two have
* therefore, (x y z) is either (+ – +) or (– + –)
that sort of reasoning will be an excellent start. from there, there's no telling which way the wind will blow – just study your number properties, and you should be able to figure out the rest.
oh yeah, you should avoid 'solving' if at all possible: you should try to think in the abstract about the signs of the numbers, and about the situation resulting from each possible combination of signs. if that sort of reasoning gets you nowhere, *then* try plugging in numbers and solving as plan b.

49.
Well Z could be –1 and n could be any even integer. Then the result is 1. So from (1) we can say that Z is either 1 or –1. So the correct answer should be C.
For z^n = 1, and n being a non zero integer, there are 3 possible ways.
a. 1^1 = 1
b. 1^– 1 = 1
c. – 1^2 = 1
Statement 1 not conclusive. Z could be 1 or –1.
Statement 2: z > 0.

==> we need both statements to solve for z.
Answer: C.

50.
Answer is D

the first thing you should do here is rephrase the question.
big takeaway:
if you see the ABSOLUTE VALUE OF A DIFFERENCE, you should recast it as the

DISTANCE BETWEEN THE TWO THINGS on the number line.

therefore, |y – a| is the distance between y and a, and so on.

hence:

QUESTION: is y closer to a than to b ?
(1) z is closer to a than to b
(2) y is closer to a than z is to b

statement (1):
note that the distance y–a is less than the distance z–a, because y is placed between a and z.
also, note that the distance y–b is greater than the distance z–b, since z lies between y and b.
therefore:
distance y–a < distance z–a < distance z–b < distance y–b (note that these are color–coded to the statements above)
so, distance y–a < distance y–b
"yes"
SUFFICIENT

statement (2):
same thing as statement (1), except for the second term of the inequality above isn't there anymore.
i.e., distance y–a < distance z–b < distance y–b.
SUFFICIENT

ans = (d)

OR

First, let's try to clear the absolute values.

Because we know that a < y < z < b, we know
abs(y – a) = y – a
abs(y – b) = b – y (since y – b is negative)

We can rephrase the question:
Is y – a < b – y?
or
Is 2y < a + b?

Statement (1) can be rephrased: z – a < b – z, so 2z < a + b. We also know that since y < z, then 2y < 2z. So 2y < 2z < a + b. (1) is sufficient.

Statement (2) can be rephrased: y – a < b – z, so y + z < a + b. Since y < z, we can add y to both sides: $2y < y + z$. So $2y < y + z < a + b$, so (2) is sufficient as well.

51.
Answer: E.
Specifically, rounding rounds ALL 5's *up*, ALL the time, even if they're followed by nothing at all.
This is why the problem doesn't contain the word "round": according to traditional rounding, 4.5 rounds to 5

the wording in the actual problem, though, is completely unambiguous: "4 is *the* integer that is closest to $x + y$".
this statement actually rules out BOTH 3.5 and 4.5, because each of those numbers is *equidistant* from two integers: the former from 3 and 4, and the latter from 4 and 5.

therefore, here are the CORRECT rephrases:
(1) $3.5 < x + y < 4.5$
(2) $0.5 < x - y < 1.5$

all four of those signs are strict inequalities. there are no $\leq$'s or $\geq$'s in this problem.

you can add all 3 corresponding parts of the inequalities directly:
$3.5 < x + y < 4.5$
$0.5 < x - y < 1.5$

$4 < 2x < 6$

therefore
$2 < x < 3$

notice that all this discussion of $<$'s, $\leq$'s, $>$'s, and $\geq$'s is immaterial in the final analysis, because there are still numbers greater than 2.5 (which are closest to 3) and numbers less than 2.5 (which are closest to 2). therefore, insufficient even if you misinterpret the question prompt as referring to "rounding".
but they *could*, easily, write a problem that would turn on the inclusion/exclusion of a number such as 4.5.

here's an example:
what number results if the number x is rounded to the nearest hundred?
(1) the multiple of 20 that is closest to x is 140.
(2) x is within ten units of 140.
here, statement (1) means that $130 < x < 150$. that's a strict inequality, which doesn't apply to 130 and 150 themselves (since 130 is just as close to 120 as to 140, and 150 is just as close to 160 as to 140).
all of these numbers give 100 when rounded to the nearest hundred, so this statement is

sufficient.
statement (2), on the other hand, means that $130 \leq x \leq 150$. this inequality includes 130 and 150.
since 150 rounds to 200, this statement is insufficient.
in this problem, the inclusion vs. exclusion of 150 makes all the difference.

52.
you can ADD TWO INEQUALITIES TOGETHER if the inequalities are BOTH "<" OR BOTH ">".

(you can't add them if one is "<" and the other is ">". if that's the situation, then you should multiply one of the inequalities by –1, or just turn it around, so that both of them are either "<" or ">".)

$-q>n-p$
+
$q>p$
____
$0>n$

therefore, C.

53.
statement (1)

all we know is that z^3 is AN INTEGER. in particular, we can't deduce that z^3 is a perfect cube.

if z^3 is a PERFECT CUBE, such as 1, 8, or 27, then z will be an integer.
if z^3 is NOT a perfect cube, such as 2, 3, 4, etc., then z will NOT be an integer.

therefore, INSUFFICIENT.

(notice that you can easily find this by PLUGGING IN NUMBERS. in fact, the very first two positive integers, 1 and 2, give "yes" and "no" respectively, so that's a clear "insufficient".)

if we assume that z^3 is a perfect cube, then we're assuming that z is an integer. if we make that (totally unfounded) assumption, then we shouldn't be surprised when we find a specious answer of "yes".

—

statement (2) is insufficient

—

together is actually SUFFICIENT.

* consider all the numbers that satisfy statement (2):
1/3, 2/3, 1, 4/3, 5/3, 2, etc.

* of these, the only ones that satisfy statement (1) as well are 1, 2, 3, ...
(all the fractional ones will still be fractions when you cube them)

* since these – the numbers that satisfy BOTH statements – are all integers, we have TOGETHER = SUFFICIENT.

answer = (c)

54.
(2) tells us that the magnitude of x is more than the magnitude of y and x and y are either both positive or both negative.
If x and y are both positive, x has to be more than y… if x and y are both negative, x has to be less than y (-2 is less than -1).
(1) tells us that x – y = ½… so x has to be more than y…
Combining, x and y are both +ve. Ans. C

55.

if you know that x > y, then you know that x – y is positive, and vice versa.
if you know that x < y, then you know that x – y is negative, and vice versa.

it's not hard to manipulate to get these statements; for instance, merely subtracting y from both sides of x > y will give x – y > 0.
but that's not the point; the point is to *recognize, INSTANTLY*, that knowing the status of the *inequality*involving x and y (i.e., whether x > y or x < y) is equivalent to knowing the sign of x – y.

well, the question prompt is:
is (m – k)(x – y) > 0?

based on the considerations above, statement #1 gives us the sign of the expression (m – k), and statement #2 gives us the sign of the expression (x – y).
if we have both of these signs, then we can figure out the sign of their product, so both statements together are sufficient.
(note that we don't even have to figure out the actual signs; it's good enough to *realize that we can find them*)

so, should be (c)

OR

mx + ky > kx + my
mx – kx > my – ky
(m–k)x > (m – k)y
(m–k)x – (m–k)y > 0
(m–k)(x–y) > 0
ie m > k and x > y

Answer C

56.
"500 is the multiple of 100 that is closest to X"

this means that, of all multiples of 100, 500 comes closest to x.
in other words, 500 is closer to x than is 100, 200, 300, 400, or 600, 700, 800, ...
if you think about this for a sec, you'll realize that it means x has to be strictly between 450 and 550.

Since the numbers don't have to be integers, you have
1. 450 < x < 550 (excluding BOTH endpoints) – note that x could be 450.00001 or 549.99999
2. 350 < y < 450 (again excluding both endpoints)

also, watch your <'s and ≤'s.

Range of X: 450<X<550
Range of Y: 350<X<450

By 1: Say X=499
Now if Y = 449 then nearest multiple of 100 to X+Y will be: 900
Say X=449 Now if Y = 350 then nearest multiple of 100 to X+Y will be: 800
Similarly you can prove that it is E

57.
Here are the TWO DEADLY ASSUMPTIONS:
1. NEVER assume that numbers are integers, unless you're told, or can infer, that they are.
2. NEVER assume that numbers are positive, unless you're told, or can infer, that they are.

There are numbers between –2 and –1, and those numbers are precisely the reason why the answer to this problem is (e).

58.

IS $1 / p > [r / (r^2 + 2)]$
St 1. case 1 P=r=2

a=1/p = 0.5 and b=r/(r^2+2) = 1/3 = 0.33

a>b

case 2 p=r=-2

a=1/p = 1/-2 = -0.5 and b= r/(r^2+2) = -1/3 = -0.33

a < b

not suff

St 2 don't know anything about p not sufficient

combine both, as shown in statement 1 whenever p or r > 0, a > b. sufficient

answer C

59.
8/9 + 1/8 = 73/72, which is greater than 1.
therefore, knowing that x + y < 73/72 is insufficient to address the issue of whether x + y < 1, because x + y could be, say, 1/2 ("yes") or any value *between* 1 and 73/72 ("no").

therefore, (e).

60.
statement (1):

(a + b)/(a – b) < 0
therefore, a + b and a – b have opposite signs. we can divide this statement into 2 cases.

CASE 1: a + b is positive and a – b is negative
a – b is negative —> immediately know **a < b**
also, in this case, a + b > a – b, so therefore b > –b, so therefore **b is positive**.
that's all we know, though; we know nothing about the sign of a. (note that this case works for (a, b) = (2, 4) but also (–2, 4))

CASE 2: a + b is negative and a – b is positive
a – b is positive —> immediately know **a > b**
in this case, a + b < a – b, so therefore b < –b, so therefore **b is negative**.
again, that's all we know. (this case works for (a, b) = (2, –4) but also (–2, –4))

this is insufficient, because there's a case in which a < b and a case in which a > b.

statement 2:
obviously insufficient

together:
this has to be CASE 2 above, so therefore a > b.
sufficient.

61.
The first step is rephrase the question. IS my=rx?
Statement 1 states that m/y = x/r
This does not help us to calculate as to whether m/r is equal to x/y. So, MAYBE!
(INSUFFICIENT)
Statement 2 states m+x/r+y = x/y
You can cross multiply rewriting the equation as y(m+x) = x(r+y) ———> my+yx = rx+yx
Subtracting yx from both sides, the equation then becomes my=rx, which is the rephrase of the question itself. SUFFICIENT.
B is the answer.

* **if you don't know what else to do with a proportion, cross–multiply it.**
(1) the reason this is valuable is because there are all sorts of versions of the same proportion that LOOK different as proportions, but which are shown to be the same when cross–multiplied. for instance, ALL of the following proportions
a/b = c/d
a/c = b/d
d/b = c/a
d/c = b/a
are equivalent, as all of them multiply to give ad = bc (as do countless others, such as (a + c)/(b + d) = c/d, after cancellation).
(2) **this applies only to proportions with EQUALS SIGNS** in them, **NOT to inequalities**. if you have an inequality such as a/b < c/d, then you can't cross–multiply it unless you know the sign of the product of the two denominators, bd (because that's all cross multiplication is: multiplying by both denominators at once on both sides). if bd is positive, then the sign won't flip; if bd is negative, then the sign must flip.

62.
(1) is surely enough.
if you have a simultaneous equation and inequality, then solve the equation and then substitute it into the inequality.

Correct answer is D

2x+5y= 20
Or, y= (20–2x)/5

–2x>3y

Substiture for y
–10x>60–6x
–4x>60

The only way this would be possible is if x is negetive
Hence 2 is sufficient

63.
note that the expression c + d appears in the question prompt. therefore, solve for this expression in statement 2: c + d = 300 – b.
now, substitute this into the question prompt, and also substitute a + b = 200:
is 200 > 300 – b ?
rephrase by solving —> **is b > 100?**

thus, it still comes down to observation that b must be more than 100, because it's the larger one of two numbers that add to 200 and therefore must be greater than half of 200.
but if you rephrase the question in this way, it's much more clear that you actually have to *think* about whether b > 100.

TAKEAWAY:
takeaway: if 2 numbers add up to n, then the larger number is more than n/2, and the smaller number is less than n/2.

it is given a+b = 200, a < b , is a+b > c+d
rephrase the question
we know a+b = 200, so
is 200 > c+d or is c+d < 200 ? This is a YES/NO question

1. c+d < 200 (Sufficient)
2. b + c +d = 300 – eq 1
add a to both the sides
a + b + c + d = 300 + a
we know a + b = 200, so
200 + c + d = 300 + a
c + d = a + 100
Now we know that a < b , a was equal to b a would be 100 and b would be 100, thus since a < b, a < 100
therefor a + 100 < 200 and c + d < 200 — > Sufficient

Answer D

64.
when you consider a problem like this, in which you are GIVEN INEQUALITIES, you should always CONSIDER THE EXTREMES of the given inequalities.

this technique is very simplistic, yet very powerful: consider the extremes to find the extremes. therefore, it's sufficient to think about, say, 0.1 and 0.9 for r, and 1.1 and 1.9 for s.

statement (i): 0.1/1.1, 0.9/1.1, 0.1/1.9, and 0.9/1.9 are all less than 1, so you're good.
statement (ii): works for (0.1)(1.1), (0.9)(1.1), and (0.1)(1.9), but NOT (0.9)(1.9).
statement (iii): only works for 1.1 – 0.9, doesn't work for any of the other pairs.

notice that this method is systematic: you don't just generate numbers at random, you generate numbers at the extremes of the intervals dictated by the inequality/ies.

I. Any number between 0 & 1 divided by any number between 1 & 2, will always be < 1
II. 2 cases: Consider r = 0.9 and s = 1.5, rs = 1.35. Consider r = 0.1 and s = 1.1, then rs = 0.11, so not true
III. 2 cases: 1.9 – 0.1 = 1.8 (this is > 1), 1.1 – 0.9 = 0.2 (< 1)

hence only I

65.
statement (1):
there's a statement called the pigeonhole principle, which basically says the following two things:
* if the AVERAGE of a set of integers is an INTEGER n, then at least one element of the set is $\geq$ n.
* if the AVERAGE of a set of integers is a NON–INTEGER n, then at least one element of the set is $\geq$ the next integer above n.
this principle is easy to prove: if you assume the contrary, then you get the absurd situation in which every element of a set is below the average of the set. that is of course impossible.

specifically, statement (1) is a case of the first part of the principle: the average of the set is 6/3 = 2, so at least one element of the set must be 2 or more.
again, you can prove this by reductio ad absurdum: if no one had sold 2 or more tickets, then you'd have a set in which everyone sold either 0 or 1 ticket, but the average is somehow still 2. that's untenable.

—

statement (2):
there are only two ways not to sell at least 2 tickets: sell 0 tickets, and sell 1 ticket.
if everyone sells a different # of tickets, then you can't fit three people into these two categories. therefore, someone must have sold at least 2 tickets.

(1) if they sold 6 together, the possibilities (2,2,2), (1,2,3), (0,3,3) (different variations of these). In all cases, there is at least one with 2 or more.

(2). This I think is real cool.. if one of them is 0, the other is 1, the third one has to be 2 or more, hence sufficient.

Hence the answer is D.

66.
we have the equation $zt < -3$, and it wants to know whether $z < 4$.

(1) alone: If $z < 9$, then we still don't know whether $z < 4$. (For instance, z could be 3 [yes] or 5 [no].)
(2) alone: If $t < -4$, then we know that z is a positive number (because the product is negative and t is negative). you can't really divide two inequalities in any simple way, so just try plugging numbers. let t be, say, –10, and try z's that are greater than 4 and less than 4 (remember, the point here, as on all DS problems, is to prove 'MAYBE'). make sure that you don't violate the condition $zt < -3$.

if $t = -10$ and $z = 1$, then the condition $zt < -3$ is satisfied, and z is not < 4.
if $t = -10$ and $z = 5$, then the condition $zt < -3$ is satisfied, and $z < 4$.
these two results show that (2) alone is insufficient.
in fact, the same two results show that statements (1) and (2) TOGETHER are insufficient (since both of the z's selected here happen to be < 9).

answer = e

OR
The answer is E.

It is a YES/NO question whether $z < 4$

Looking at it 1st statement, for the 1st inequality to be true either z is –ve or t –ve, however both cannot be negative.

1. $z < 9$ – Insufficient as $z = 8$, $t = -9$ then $zt < -3$, similarly $z = 1$ and $t = -5$, $zt < -3$ thus it is insufficient to state that $z <4$

2. $t < -4$ Insufficient since this only provides us that $z > 0$, but does not given any indication whether $z > 4$
for example $t = -6$, $z = 2$, $zt = -12$
$t = -5$, $z = 5$ $zt = -25$

Now taking them together: $z < 9$ and $t < -4$
If one has to be negative, for the 1st statement to hold true than z is +ve between 1 and 9 and t –ive –4 to –infinity.
$z = 8$, $t = -5$; $zt = -40$

z = 3, t = –5 ; zt = –15
Thus one cannot conclusively state whether z< 4, therefore the correct answer is E.

If you simplify the stem as below:
zt < –3
z < –3/t
and since t < –4 from Statement 2:
if t = –5 —> z < –3/–5 = 0.6 —> z < 0.6 is less than 4
if t = –10 —> z < –3/–10 = 0.3 —> z < 0.3 which is less than 4

WRONG…
you can't divide by t unless you've ascertained whether t is positive or negative. and moreover, if t is negative, then you have to switch around the inequality sign ("<" becomes ">").
so if t < –4, then you actually have z **>** –3/t, not "<".

67.
remember that if c is positive, then the statement is guaranteed to be true (because b is already greater than a, so adding something positive will keep it that way).

statement (1)
in this case, you know that each of b and c is greater than a, but that's all you know.
if b and c are positive, then this is good enough, because then b + c will be greater than either b or c (both of which are already greater than a). so that's a Yes.
with negative values, though, you can get a No. if b = –2, c = –3, and a = –4, then b > a and c > a, but b + c < a. that's a No.
insufficient.

(2) means one of the following: (a) all three are positive, (b) two are negative and one is positive.
if all 3 are positive, then *a fortiori* c is positive, so b + c must be greater than a.
but
if b is positive, and a and c are negative, then it's possible that b + c is not greater than a. if you don't like plugging actual numbers, then consider the idea that c could be a REALLY BIG NEGATIVE that cancels out the positive–ness of b. for instance, if a = –1, b = 2, and c = –100, then b > a, but b + c <<<< a.
insufficient.

(together)
in this case, a is the smallest of the 3 numbers.
3 cases:
* all positive: this is a yes, as established before
* a < b < 0 < c: this is also a yes, because c is positive
* a < c < 0 < b:
rearrange the inequality to "is b > a – c ?"

in this case, notice that b is positive and a – c is negative, so this is still a yes.
always yes
sufficient

ans = c

68.
OA is B

the best way to solve this problem is to notice that its subject matter is POSITIVES AND NEGATIVES. how do you know this? because it deals *only* with absolute values and inequality signs – no other numbers or non–absolute values in sight to mess things up.

there is no way to 'quickly solve' this one algebraically, unfortunately. in fact, even at the highest levels of mathematics, the best (and really the only) way to solve problems like these is case–wise, considering the different possibilities for + and – one case at a time.

A) y < x
a case for >
x = 2, y = –3
LHS = 5 > RHS = –1
Case for = or <
x = anything, y = 0
LHS = RHS
Insufficient

B) xy < 0. x and y are on opposite sides of 0 on the number line
| x – y | – distance of x from y
|x| – distance of x from 0
|y| – distance of y from 0
If you imagine a number line
like this
x————0—————y
or
y————0—————x

you can conclude that the distance between x and y is greater than the difference between x,0 and y,0.

OR

Lets take (1)
y<x

y 2 –4

x 4 2

Substitute in the equation

$|x-y|>|x|-|y|$

Using the above values,
$|2|>|4|-|2|$ First Column Values

Not true
$|2-(-4)|>|2|-|-4|$ Second Column Values
$|6|>2-4$
$6>-2$ —True

Hence A is Insuff

(2)
$xy<0$ which means either x or y should be negative

x 7 –7
y –3 3

Substitute the values in the eq
$|x-y|>|x|-|y|$

$|10|>|7|-|-3|$
$10>4$

$|-7-3|>|-7|-|3|$
$|-10|>-7-3$
$10>-10$

Hence Suff

Hence B.

69.
REPHRASE
is y <1
statement 1) doesnt tell us anything about Y
statement 2) if y<0 then y must be <1
B is the answer

70.
Ans: E

1) 7x–2y>0

7(8)–2(3) = positive
7(8)–2(–3) = positive

y can be neg or pos. NOT sufficient.

2) –y<x

x = 8 , y can equal 3
x = 8, –3<8, so y can be positive

x = 8, y can equal –3
x=8, –(–3)<8, 6<8, y can be negative too. Not sufficient.

c) combined: Look at the breakdown from statement (1) with numbers that also satisfy statement (2)'s criteria. y can be neg OR positive.

Answer is Choice E.

71.

1. If that is that case.

   i) is insufficient. there is not enough information on the values of x,y,z.
   ii) is sufficient. there are 3 possibilities: x<y=z, x=y=z, y=z<x. in any case z is the median.

   Hence the answer is B

If you need convincing about answer a, then remember that your goal on these types of problems is to try to prove 'maybe' (i.e., insufficient). so try to find 2 groups of numbers, one of which gives a 'yes' answer and one of which gives a 'no' answer:
(x, y, z) = (1, 3, 5): z is not the median
(x, y, z) = (1, 5, 3): z is the median
insufficient.

important note:
make sure that you understand that this is the direction of the logic in all data sufficiency questions. you are always trying to prove/disprove the prompt question, based on the evidence given in the 2 statements. you have written the question backwards – your 'if' and 'then' construct a logic that runs in precisely the opposite direction – which will make it essentially impossible for you to answer questions correctly.

—

treatment of the question:

rephrase of initial question:
this is the same as asking: **is z equal to the middle number of the three numbers?**

statement (1)
this statement tells nothing about the order of the three numbers. it could be true regardless of the order of the 3 numbers, and, more to the point, regardless of the position of z in the ordered list.
examples:
x = 1, y = 2, z = 3: z is not the median
x = 1, y = 3, z = 2: z is the median
insufficient

statement (2)
if y and z are equal, there are three possibilities:
–– they are the two largest #s in the list. in this case, both of them equal the median of the list.
–– they are the two smallest #s in the list. in this case, both of them equal the median of the list.
–– all three numbers in the list are the same. in this case, all of them equal the median.
in any of these cases, z is the median.
sufficient

answer = b

72.
OA = E
try to draw out the cases. for example

(1) xyz < 0 and |xy| > |xz| . this implies the following are possible (with the Y–axis indicated as the vertical bar)

yx | z
z | xy
xz | y
yz | x

(2) xy < 0 means that x & y are on opposite sides of the vertical axis. and |xy| > |xz|

y | zx
y | xz
x | zy

the cases illustrate the answer

1. $xyz < 0$ — so either all three or one of the three is negative
2. $xy < 0$ — either x or y is negative, BUT not both.

Scenario A) where $XYZ < 0$.

2 positives 1 negative
3 negatives.

For three negatives on the number the order can be z, x, y, 0...or x, z, y, 0.

So A is insufficient.

Scenario B) hwere $XY < 0$.

One negative one positive. So the order can go. x 0 z y...or z x 0 y. So B is insufficient.

Scenario A+B.

If X and Y are both of oppositie signs ($XY<0$)..then Z has to be postive in order for $XYZ < 0$.

So One Negative and two positives.

Order can be.....x 0 z y...or y 0 x z.

Thus E.

73.
$x + y = 1$

If $y>=0.15$, then try some:
If $y=0.15$, $x = 0.85$ in which case, no, x is not less than 0.8.
If $y=0.3$, then $x=0.7$, in which case, yes, x is less than 0.8.
Contradictory answers = indufficient.

$y = 1–x$
If $C >=7.30$, then try some:
If $C=7.30$, then $7.3 = 6.5x + 8.5(1–x)$
$7.3 = 6.5x + 8.5 – 8.5x$
$–1.2 = –2x$
$1.2/2 = x$
$x = 0.6$, so yes, $x < 0.8$.

If x+y=1, and I need to create a larger C, I have to make x smaller and y larger (because I multiply y by 8.5 in the formula while only multiplying x by 6.5). So, x will decreas as C increases. As a result, I can always say that $x < 0.8$. Sufficient.

74.
The correct answer is D

(1) if m<p, mv<pv<0 will only hold if V is +ve and M and P are –ve. For other two cases that you mentioned mv>pv so they are not valid.

Hence, for mv<pv<0 V to hold, V has to +ve. So (1) is sufficient.
Luci, you were on the right track with your process, but you didn't actually work out the numbers, so you didn't notice that some of them were inconsistent with the given condition (mv<pv<0). I love trying numbers as a technique, but make sure you follow through just a bit more on the calculations.

If you had, you would have seen:
ex. m=3, p=5, and let's make v =–2
mv = –6
pv = –10
–6<–10<0
Which is not true, so that combination is invalid – I can't use it to test the statement. The only way to make it work would be to make p less than m, but statement one gives the condition m<p, so I can't do it.

75.
E

DS Tricky Questions

1.

For this overlapping set problem, we want to set up a two-set table to test our possibilities. Our first set is vegetarians vs. non-vegetarians; our second set is students vs. non-students.

	VEGETARIAN	NON-VEGETARIAN	TOTAL
STUDENT			
NON-STUDENT		15	

TOTAL	x	x	?

We are told that each non-vegetarian non-student ate exactly one of the 15 hamburgers, and that nobody else ate any of the 15 hamburgers. This means that there were exactly 15 people in the non-vegetarian non-student category. We are also told that the total number of vegetarians was equal to the total number of non-vegetarians; we represent this by putting the same variable in both boxes of the chart.

The question is asking us how many people attended the party; in other words, we are being asked for the number that belongs in the bottom-right box, where we have placed a question mark.

The second statement is easier than the first statement, so we'll start with statement (2).

(2) INSUFFICIENT: This statement gives us information only about the cell labeled "vegetarian non-student"; further it only tells us the number of these guests as a *percentage* of the total guests. The 30% figure does not allow us to calculate the actual number of any of the categories.

(1) SUFFICIENT: This statement provides two pieces of information. First, the vegetarians attended at the rate, or in the ratio, of 2:3 students to non-students. We're also told that this 2:3 rate is half the rate for non-vegetarians. In order to double a rate, we double the first number; the rate for non-vegetarians is 4:3 We can represent the actual numbers of non-vegetarians as $4a$ and $3a$ and add this to the chart below. Since we know that there were 15 non-vegetarian non-students, we know the missing common multiple, a, is 15/3 = 5. Therefore, there were (4)(5) = 20 non-vegetarian students and 20 + 15 = 35 total non-vegetarians (see the chart below). Since the same number of vegetarians and non-vegetarians attended the party, there were also 35 vegetarians, for a total of 70 guests.

	VEGETARIAN	NON-VEGETARIAN	TOTAL
STUDENT		$4a$ or 20	
NON-STUDENT		$3a$ or 15	
TOTAL	x or 35	x or 35	? or 70

The correct answer is A.

2.

For an overlapping set question, we can use a double-set matrix to organize the information and solve. The two sets in this question are the practical test (pass/fail) and the written test (pass/fail).

From the question we can fill in the matrix as follows. In a double-set matrix, the sum of the first two rows equals the third and the sum of the first two columns equals the third. The bolded value was derived from the other given values. The question asks us to find the value of $.7x$

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	**$.7x$**	$.3x$	x
WRITTEN - FAIL		0	
TOTALS		$.3x$	

(1) INSUFFICIENT: If we add the total number of students to the information from the question, we do not have enough to solve for $.7x$.

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	**$.7x$**	$.3x$	x
WRITTEN - FAIL		0	
TOTALS		$.3x$	188

(2) INSUFFICIENT: If we add the fact that 20% of the *sixteen year-olds who passed the practical test* failed the written test to the original matrix from the question, we can come up with the relationship $.7x = .8y$. However, that is not enough to solve for $.7x$.

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	**$.7x = .8y$**	$.3x$	x
WRITTEN - FAIL	$.2y$	0	$.2y$
TOTALS	y	$.3x$	

(1) AND (2) SUFFICIENT: If we combine the two statements we get a matrix that can be used to form two relationships between x and y:

	PRACTICAL - PASS	PRACTICAL - FAIL	TOTALS
WRITTEN - PASS	**$.7x = .8y$**	$.3x$	x
WRITTEN - FAIL	$.2y$	0	$.2y$
TOTALS	y	$.3x$	188

$.7x = .8y$
$y + .3x = 188$

This would allow us to solve for x and in turn find the value of $.7x$, the number of sixteen year-olds who received a driver license.

The correct answer is C.

3.
Let i be the salesman's income, let s be the salary, and let c be the commission. From the question stem we can construct the following equation:
$i = s + c$
We are asked whether s accounts for more than half of i. We can thus rephrase the question as "Is s greater than c?"

SUFFICIENT: This allows us to construct the following equation:
$1.1i = s + 1.3c$
Since we already have the equation $i = s + c$, we can subtract this equation from the one above:
$.1i = .3c$
Notice that the s's cancel each other out when we subtract. We can isolate the c by multiplying both sides by 10/3 (the reciprocal of .3 or 3/10):
$(1/10)i = (3/10)c$
$(1/10)i \times (10/3) = (3/10)c \times (10/3)$
$(1/3)i = c$
Therefore c is one-third of the salesman's income. This implies that the salary must account for two-thirds of the income. Thus, we can answer definitively that the salary accounts for more than half of the income.

INSUFFICIENT: Either $s - c = .5s$ or $c - s = .5s$. Coupled with our knowledge that s and c must add to 100% of the salesman's income, we can say that one of the two is worth 75% of the income and the other is worth 25%. However, we don't know which is the bigger number: s or c.

The correct answer is A.

4.
In order to determine the percent discount received by Jamie, we need to know two things: the regular price of the desk and the sale price of the desk. Alternatively, we could calculate the percent discount from the price reduction and either the regular price or the sale price.

(1) INSUFFICIENT: This statement tells us the regular price of the desk at Al’s, but provides no information about how much Jamie actually paid for the desk during the annual sale.

(2) INSUFFICIENT: This statement tells us how much the price of the desk was reduced during the sale, but provides no information about the regular price. For example, if the regular price was \$6010, then the discount was only 10%. On the other hand, if the regular price was \$602, then the discount was nearly 100%.

(1) AND (2) INSUFFICIENT: At first glance, it seems that the statements together provide enough information. Statement (1) seems to provide the regular price of the desk, while statement (2) provides the discount in dollars.

However, pay attention to the words "rounded to the nearest percent" in statement (1). This indicates that the regular price of the desk at Al's is 60% of the MSRP, plus or minus 0.5% of the MSRP. Rather than clearly stating that the regular price is (0.60)($2000) = $1200, this statement gives a range of values for the regular price: $1200 plus or minus $10 (0.5% of 2000), or between $1190 and $1210.

If the regular price was $1190, then the discount was ($601/$1190) × 100% = 50.5% (you can actually see that this is greater than 50% without calculating).

If the regular price was $1210, then the discount was ($601/$1210) × 100% = 49.7% (you can actually see that this is less than 50% without calculating).

The uncertainty about the regular price means that we cannot answer with certainty whether the discount was more than 50% of the regular price.

The correct answer is E.

5.
For 1, the tip for a $15 bill will be $2, which is less than $15*15%=2.25; the tip for a $20 will be $4, which is greater than $15*15%=2.25. Insufficient.
For 2, tips is $8, means the tens digit of the bill is 4, and the largest possible value of the bill is $49. $8>49*15%=7.35. Sufficient alone.
Answer is B

6.
To find the combined rate of Machines A and B, we combine their individual rates. If Machine A can fill an order of widgets in *a* hours, then in 1 hour it can fill $\frac{1}{a}$ of the order. By the same token, if Machine B can fill the order of widgets in *b* hours, then in 1 hour, it can fill $\frac{1}{b}$ of the order. So together in 1 hour, Machines A and B can fill $\frac{1}{a} + \frac{1}{b}$ of the order:

$$\frac{1}{a} + \frac{1}{b} = \frac{(b)1}{(b)(a)} + \frac{(a)1}{(a)(b)} = \frac{b}{ab} + \frac{a}{ab} = \frac{a+b}{ab}$$

So in 1 hour, Machines A and B can complete $\frac{a+b}{ab}$ of the order. To find the number of hours the machines need to complete the *entire* order, we can set up the following equation:

(fraction of order completed in 1 hour) x (number of hours needed to complete entire order) = 1 order.

If we substitute $\frac{a+b}{ab}$ for the fraction of the order completed in 1 hour, we get:

$\frac{a+b}{ab}(x) = 1$, where *x* is the number of hours needed to complete the entire order. If we divide both sides by $\frac{a+b}{ab}$, we get:

$$x = \frac{ab}{a+b}$$

In other words, it will take Machines A and B $\frac{ab}{a+b}$ hours to complete the entire order working together at their respective rates.

The question stem tells us that *a* and *b* are both even integers. We are then asked whether *a* and *b* are equal. If they are equal, we can express each as $2z$, where *z* is a non-zero integer, because they are even. If we replace *a* and *b* with $2z$ in the combined rate, we get:

$$\frac{(2z)(2z)}{2z+2z} = \frac{4z^2}{4z} = z$$

So if *a* and *b* are equal, the combined rate of Machines A and B must be an integer (since *z* is an integer). We can rephrase the question as:

Is the combined rate of Machines A and B an integer?

Statement 1 tells us that it took 4 hours and 48 minutes for the two machines to fill the order (remember, they began at noon). This shows that the combined rate of Machines A and B is NOT an integer (otherwise, it would have taken the machines a whole number of hours to complete the order). So we know that *a* and *b* cannot be the same. Sufficient.

Statement 2 tells us that $(a+b)^2 = 400$. Since both *a* and *b* must be positive (because they represent a number of hours), we can take the square root of both sides of the equation without having to worry about negative roots. Therefore, it must be true that $a + b = 20$. So it is possible that $a = 10$ and that $b = 10$, which would allow us to answer "yes" to the question. But it is also possible that $a = 12$ and $b = 8$ (or any other combination of positive even integers that sum to 20), which would give us a "no". Insufficient.

The correct answer is A: Statement 1 alone is sufficient, but statement 2 alone is not.

7. If water is rushing into tank 1 at x gallons per minute while leaking out at y gallons per minute, the net rate of fill of tank 1 is $x - y$. To find the time it takes to fill tank 1, divide the capacity of tank 1 by the rate of fill: $z / (x - y)$.

We know that the rate of fill of tank 2 is y and that the total capacity of tank 2 is twice the number of gallons remaining in tank 1 after one minute. After one minute, there are $x - y$ gallons in tank 1, since the net fill rate is $x - y$ gallons per minute. Thus, the total capacity of tank 2 must be $2(x - y)$.

The time it takes to fill tank two then is $\dfrac{2(x-y)}{y}$.

The question asks us if tank 1 fills up before tank 2.

We can restate the question: Is $\dfrac{z}{x-y} < \dfrac{2(x-y)}{y}$?

SUFFICIENT: We can manipulate $zy < 2x^2 - 4xy + 2y^2$:

$zy < 2x^2 - 4xy + 2y^2$
$zy < 2(x^2 - 2xy + y^2)$
$zy < 2(x - y)(x - y)$ (dividing by $x - y$ is okay since $x - y > 0$)
$\dfrac{zy}{x-y} < 2(x - y)$ (dividing by y is okay since $y > 0$)
$\dfrac{z}{x-y} < \dfrac{2(x-y)}{y}$

This manipulation shows us that the time it takes to fill tank 1 is definitely longer than the time it takes to fill tank 2.

INSUFFICIENT: We can express this statement algebraically as: $1/2(z) > 2(x - y)$. We cannot use this expression to provide us meaningful information about the question.

The correct answer is A.

8.

This question cannot necessarily be rephrased, but it is important to recognize that we need not necessarily calculate Wendy's or Bob's travel time individually. Determining the difference between Wendy's and Bob's total travel times would be sufficient. This difference might be expressed as $t_b - t_w$.

(1) INSUFFICIENT: Calculating Bob's rate of speed for any leg of the trip will not give us sufficient information to determine the time or distance of his journey, at least one of which would be necessary to determine how quickly Wendy reaches the restaurant.

(2) SUFFICIENT: To see why this statement is sufficient, it is helpful to think of Bob's journey in two legs: the first leg walking together with Wendy (t_1), and the second walking alone (t_2). Bob's total travel time $t_b = t_1 + t_2$. Because Wendy traveled halfway to the restaurant with Bob, her total travel time $t_w = 2t_1$. Substituting these expressions for $t_b - t_w$,

$t_1 + t_2 - 2t_1 = t_2 - t_1$

$t_b - t_w = t_2 - t_1$

Statement (2) tells us that Bob spent 32 more minutes traveling alone than with Wendy. In other words, $t_2 - t_1 = 32$. Wendy waited at the restaurant for 32 minutes for Bob to arrive.

The correct answer is B.

9.

Let's say:
I = the original amount of bacteria
F = the final amount of bacteria
t = the time bacteria grows
If the bacteria increase by a factor of x every y minutes, we can represent the growth of the bacteria with the equation:
$F = I(x)^{t/y}$

To understand why, let's assign some values to I, x and y:

I =	100
x =	2
y =	3

If the bacteria start off 100 in number and they double every 3 minutes, after 3 minutes there will be 100(2) bacteria. Let's construct a table to track the growth of the bacteria:

t (time)	F (final count)
3	$100(2) = 100(2)^1$
6	$100(2)(2) = 100(2)^2$
9	$100(2)(2)(2) = 100(2)3$
12	$100(2)(2)(2)(2) = 100(2)^4$

We can generalize the F values in the table as $100(2)^n$.
The 100 represents the initial count, I.
The 2 represents the factor of growth (in this problem x).
The n represents the number of growth periods. The number of growth periods is found by dividing the time, t, by the amount of time it takes to complete a period, y.
From this example, we can extrapolate the general formula for exponential growth: $F = I(x)^{t/y}$
This question asks us how long it will take for the bacteria to grow to 10,000 times their original amount.
The bacteria will have grown to 10,000 times their original amount when $F = 10{,}000I$.
If we plug this into the general formula for exponential growth, we get: $10{,}000I = I(x)^{t/y}$ or $10{,}000 = (x)^{t/y}$.
The question is asking us to solve for t.
(1) SUFFICIENT: This statement tells us that $x^{1/y}=10$. If we plug this value into the equation we can solve for t.
$10{,}000 = (x)^{t/y}$
$10{,}000 =[(x)^{1/y}]^t$
$10{,}000 =(10)^t$
$t = 4$
(2) SUFFICIENT: The bacteria grow one hundredfold in 2 minutes, that is to say they grow by a factor of 10^2. Since exponential growth is characterize by a constant factor of growth (i.e. by x every y minutes), for the bacteria to grow 10,000 fold (i.e. a factor of 10^4), they will need to grow another 2 minutes, for a total of four minutes ($10^2 \times 10^2 = 10^4$).
The correct answer is D, EACH statement ALONE is sufficient to answer the question.

10. To determine the ratio of Chemical A to Chemical C, we need to find the amount of each in the solution. The question stem already tells us that there are 10 milliliters of Chemical C in the final solution. We also know that the original solution consists of only Chemicals A and B in the ratio of 3 to 7. Thus, we simply need the original volume of the solution to determine the amount of Chemical A contained in it.

    SUFFICIENT: This tells us that original solution was 50 milliliters. Thus, there must have been 15 milliliters of Chemical A (to 35 milliliters of Chemical B). The ratio of A to C is 15 to 10 (or 3 to 2).

SUFFICIENT: This tells us that the final solution was 60 milliliters. We know that this includes 10 milliliters of Chemical C. This means the original solution contained 50 milliliters. Thus, there must have been 15 milliliters of Chemical A (to 35 milliliter of Chemical B). The ratio of A to C is 15 to 10 (or 3 to 2).

The correct answer is D.

11.

Given woman: children=5:2
1). children: man=5:11, you agree it is insufficient
2). W<30, you also agree it alone is insufficient
Together, w:c:m = 25:10:22 (all have to be integers!) thus w=25 and m=22.
Answer is C

12.
Let Xf/Yf is the full time in Division X/Y, and Xp/Yp is part time in X/Y, X,Y, and Z are number of employees in X, Y, and Z.
X=Xf+Xp
Y=Yf+Yp
Z=X+Y
For 1, Yf/Yp<Zf/Zp, as a compensation, Xf/Xp should be greater than Zf/Zp
For2, More than ?Zf /less than ?of the Zp should be greater than Zf/Zp
Answer is D

13.
(1) SUFFICIENT: Statement(1) tells us that $x > 2^{34}$, so we want to prove that $2^{34} > 10^{10}$. We'll prove this by manipulating the expression 2^{34}.
$2^{34} = (2^4)(2^{30})$
$2^{34} = 16(2^{10})^3$

Now $2^{10} = 1024$, and 1024 is greater than 10^3. Therefore:

$2^{34} > 16(10^3)^3$
$2^{34} > 16(10^9)$
$2^{34} > 1.6(10^{10})$.

Since $2^{34} > 1.6(10^{10})$ and $1.6(10^{10}) > 10^{10}$, then $2^{34} > 10^{10}$.

(2) SUFFICIENT: Statement (2) tells us that that $x = 2^{35}$, so we need to determine if $2^{35} > 10^{10}$. Statement (1) showed that $2^{34} > 10^{10}$, therefore $2^{35} > 10^{10}$.

The correct answer is D.

14.

This question cannot be rephrased since it is already in a simple form.

(1) INSUFFICIENT: Since x^2 is positive whether x is negative or positive, we can only determine that x is not equal to zero; x could be either positive or negative.

(2) INSUFFICIENT: By telling us that the expression $x \cdot |y|$ is not a positive number, we know that it must either be negative or zero. If the expression is negative, x must be negative ($|y|$ is never negative). However if the expression is zero, x or y could be zero.

(1) AND (2) INSUFFICIENT: We know from statement 1 that x cannot be zero, however, there are still two possibilites for x: x could be positive (y is zero), or x could be negative (y is anything).

The correct answer is E.

15.

It is extremely tempting to divide both sides of this inequality by y or by the $|y|$, to come up with a rephrased question of "is $x > y$?" However, we do not know the sign of y, so this cannot be done.

(1) INSUFFICIENT: On a yes/no data sufficiency question that deals with number properties (positive/negatives), it is often easier to plug numbers. There are two good reasons why we should try both positive and negative values for y: (1) the question contains the expression $|y|$, (2) statement 2 hints that the sign of y might be significant. If we do that we come up with both a yes and a no to the question.

x	y	$x \cdot \|y\| > y^2$	?
-2	-4	$-2(4) > (-4)^2$	N
4	2	$4(2) > 2^2$	Y

(2) INSUFFICIENT: Using the logic from above, when trying numbers here we should take care to pick x values that are both greater than y and less than y.

x	y	$x \cdot \|y\| > y^2$	?
2	4	$2(4) > 4^2$	N
4	2	$4(2) > 2^2$	Y

(1) AND (2) SUFFICIENT: If we combine the two statements, we must choose positive x and y values for which $x > y$.

x	y	$x \cdot \|y\| > y^2$	?
3	1	$3(1) > 1^2$	Y
4	2	$4(2) > 2^2$	Y
5	3	$5(3) > 3^2$	Y

Using a more algebraic approach, if we know that y is positive (statement 2), we can divide both sides of the original question by y to come up with "is $x > y$?" as a new question. Statement 1 tells us that $x > y$, so both statements together are sufficient to answer the question.

The correct answer is C.

16. (1) INSUFFICIENT: This expression provides only a range of possible values for x.

(2) SUFFICIENT: Absolute value problems often -- ***but not always*** -- have multiple solutions because the expression *within* the absolute value bars can be either positive or negative even though the absolute value of the expression is always positive. For example, if we consider the equation $|2 + x| = 3$, we have to consider the possibility that $2 + x$ is already positive and the possibility that $2 + x$ is negative. If $2 + x$ is positive, then the equation is the same as $2 + x = 3$ and $x = 1$. But if $2 + x$ is negative, then it must equal -3 (since $|-3| = 3$) and so $2 + x = -3$ and $x = -5$.

So in the present case, in order to determine the possible solutions for x, it is necessary to solve for x under both possible conditions.

For the case where $x > 0$:

$x = 3x - 2$
$-2x = -2$
$x = 1$

For the case when $x < 0$:

$x = -1(3x - 2)$ We multiply by -1 to make x equal a negative quantity.
$x = 2 - 3x$
$4x = 2$
$x = 1/2$

Note however, that the second solution $x = 1/2$ contradicts the stipulation that $x < 0$, hence there is no solution for x where $x < 0$. Therefore, $x = 1$ is the only valid solution for (2).

The correct answer is B.

17. The question "Is $|x|$ less than 1?" can be rephrased in the following way.

Case 1: If $x > 0$, then $|x| = x$. For instance, $|5| = 5$. So, if $x > 0$, then the question becomes "Is x less than 1?"

Case 2: If $x < 0$, then $|x| = -x$. For instance, $|-5| = -(-5) = 5$. So, if $x < 0$, then the question becomes "Is $-x$ less than 1?" This can be written as follows:

$-x < 1$?
or, by multiplying both sides by -1, we get
$x > -1$?

Putting these two cases together, we get the fully rephrased question:
“Is $-1 < x < 1$ (and x not equal to 0)"?

Another way to achieve this rephrasing is to interpret absolute value as distance from zero on the number line. Asking "Is $|x|$ less than 1?" can then be reinterpreted as "Is x less than 1 unit away from zero on the number line?" or "Is $-1 < x < 1$?" (The fact that x does not equal zero is given in the question stem.)
(1) INSUFFICIENT: If $x > 0$, this statement tells us that $x > x/x$ or $x > 1$. If $x < 0$, this statement tells us that $x > x/-x$ or $x > -1$. This is not enough to tell us if $-1 < x < 1$.
(2) INSUFFICIENT: When $x > 0$, $x > x$ which is not true (so $x < 0$). When $x < 0$, $-x > x$ or $x < 0$. Statement (2) simply tells us that x is negative. This is not enough to tell us if $-1 < x < 1$.
(1) AND (2) SUFFICIENT: If we know $x < 0$ (statement 2), we know that $x > -1$ (statement 1). This means that $-1 < x < 0$. This means that x is definitely between -1 and 1.
The correct answer is C.

18. Note that one need not determine the values of both x and y to solve this problem; the value of product xy will suffice.

(1) SUFFICIENT: Statement (1) can be rephrased as follows:

$-4x - 12y = 0$
$-4x = 12y$
$x = -3y$

If x and y are non-zero integers, we can deduce that they must have opposite signs: one positive, and the other negative. Therefore, this last equation could be rephrased as

$|x| = 3|y|$

We don’t know whether x or y is negative, but we do know that they have the opposite signs. Converting both variables to absolute value cancels the negative sign in the expression $x = -3y$.

We are left with two equations and two unknowns, where the unknowns are $|x|$ and $|y|$:

$|x| + |y| = 32$
$|x| - 3|y| = 0$

Subtracting the second equation from the first yields

$4|y| = 32$
$|y| = 8$

Substituting 8 for $|y|$ in the original equation, we can easily determine that $|x| = 24$. Because we know that one of either x or y is negative and the other positive, xy must be the negative product of $|x|$ and $|y|$, or $-8(24) = -192$.

(2) INSUFFICIENT: Statement (2) also provides two equations with two unknowns:

$|x| + |y| = 32$
$|x| - |y| = 16$

Solving these equations allows us to determine the values of $|x|$ and $|y|$: $|x| = 24$ and $|y| = 8$. However, this gives no information about the sign of x or y. The product xy could either be -192 or 192.

The correct answer is A.

19. (1) INSUFFICIENT: Since this equation contains two variables, we cannot determine the value of y. We can, however, note that the absolute value expression $|x^2 - 4|$ must be greater than or equal to 0. Therefore, $3|x^2 - 4|$ must be greater than or equal to 0, which in turn means that $y - 2$ must be greater than or equal to 0. If $y - 2 \geq 0$, then $y \geq 2$.

(2) INSUFFICIENT: To solve this equation for y, we must consider both the positive and negative values of the absolute value expression:

If $3 - y > 0$, then $3 - y = 11$
$y = -8$

If $3 - y < 0$, then $3 - y = -11$
$y = 14$

Since there are two possible values for y, this statement is insufficient.

(1) AND (2) SUFFICIENT: Statement (1) tells us that y is greater than or equal to 2, and statement (2) tells us that $y = -8$ or 14. Of the two possible values, only 14 is greater than or equal to 2. Therefore, the two statements together tell us that y must equal 14.

The correct answer is C.

20.
We can rephrase the question: "Is $m - n > 0$?"
(1) INSUFFICIENT: If we solve this inequality for $m - n$, we get $m - n < 2$. This does not answer the question "Is $m - n > 0$?".
(2) SUFFICIENT: If we solve this inequality for $m - n$, we get $m - n < -2$. This answers the question "Is $m - n > 0$?" with an absolute NO.

The correct answer is B.

21.
Apparently, answer is D

22:
1)+2), we can know that $z>0$, then, $m>3z>0$
Together, $m+z>0$
Answer is C

23.
For 1, $x<0$, $x+|x|=0$
For 2, $y<1$, noticed that y is an integer, y only can be 0
Answer is D

24.
In order for $\frac{ab}{cd}$ to be positive, *ab* and *cd* must share the same sign; that is, both either positive or negative.
There are two sets of possibilities for achieving this sufficiency. First, if all four integers share the same sign- positive or negative- both *ab* and *cd* would be positive. Second, if any two of the four integers are positive while the other two are negative, *ab* and *cd* must share the same sign. The following table verifies this claim:

Positive Pair	Negative Pair	*ab* Sign	*cd* Sign
a, b	*c, d*	+	+
a, c	*b, d*	-	-
a, d	*b, c*	-	-
b, c	*a, d*	-	-
b, d	*a, c*	-	-

c, *d*	*a*, *b*	+	+

For the first and last cases, $\frac{ab}{cd}$ will be positive. On the other hand, it can be shown that if only one of the four

integers is positive and the other three negative, or vice versa, $\frac{ab}{cd}$ must be negative. This question can most tidily be rephrased as "Among the integers a, b, c and d, are an even number (zero, two, or all four) of the integers positive?"

(1) SUFFICIENT: This statement can be rephrased as $ad = -bc$. For the signs of ad and bc to be opposite one another, either precisely one or three of the four integers must be negative. The answer to our rephrased question is "no," and, therefore, we have achieved sufficiency.

(2) SUFFICIENT: For the product $abcd$ to be negative, either precisely one or three of the four integers must be negative. The answer to our rephrased question is "no," and, therefore, we have achieved sufficiency.

The correct answer is D.

25.

THERE IS A SLIGHT MISPRINT IN THE QUESTION:

We can first simplify the exponential expression in the question:

$b^{a+1} - ba^b$

$b(b^a) - b(a^b)$

$b(b^a - a^b)$

So we can rewrite this question then as is $b(b^a - a^b)$ odd? Notice that if either b or $b^a - a^b$ is even, the answer to this question will be no.

(1) SUFFICIENT: If we simplify this expression we get $5a - 8$, which we are told is odd. For the difference of two numbers to be odd, one must be odd and one must be even. Therefore $5a$ must be odd, which means that a itself must be odd. To determine whether or not this is enough to dictate the even/oddness of the expression $b(b^a - a^b)$, we must consider two scenarios, one with an odd b and one with an even b:

a	b	$b(b^a - a^b)$	odd/even
3	1	$1(1^3 - 3^1) = -2$	even
3	2	$2(2^3 - 3^2) = -2$	even

It turns out that for both scenarios, the expression $b(b^a - a^b)$ is even.

(2) SUFFICIENT: It is probably easiest to test numbers in this expression to determine whether it implies that b is odd or even.

b	$b^3 + 3b^2 + 5b + 7$	odd/even
2	$2^3 + 3(2^2) + 5(2)+ 7 = 37$	odd
1	$1^3 + 3(1^2) + 5(1) + 7 = 16$	even

We can see from the two values that we plugged that only even values for b will produce odd values for the expression $b^3 + 3b^2 + 5b + 7$, therefore b must be even. Knowing that b is even tells us that the product in the question, $b(b^a - a^b)$, is even so we have a definitive answer to the question.

The correct answer is D, EACH statement ALONE is sufficient to answer the question.